**Due Date: September 30, 11:59pm**

In all problems, prove the correctness of your algorithm and analyze its running time.

**Problem 1:** [10pts] Let $S = \{[s_i, f_i] : 1 \le i \le n\}$ be a set of $n$ intervals. Each interval $[s_i, f_i]$ has a weight $w_i$. A subset $R \subseteq S$ is called *feasible* if no two intervals in $R$ overlap, i.e. if $[s_i, f_i], [s_j, f_j] \in R$ then $[s_i, f_i] \cap [s_j, f_j] = \varnothing$. Describe an algorithm to find a feasible subset of $S$ of maximum weight.

**Problem 2:** Let $T_1 = \langle p_1, \ldots, p_r \rangle$ and $T_2 = \langle q_1, \ldots, q_s \rangle$ be two sequences of points in $\mathbb{R}^2$. A *correspondence* is an ordered pair of the form $(p_i, q_j)$. A set $C$ of correspondences is *monotone* if for any two correspondences $(p_i, q_j)$ and $(p_{i'}, q_{j'})$ in $C$ with $i < i'$, we have $j < j'$. Furthermore, $C$ is called an *assignment* if every point $p_i$ $(1 \le i \le r)$ and $q_j$ $(1 \le j \le s)$ appears in at least one pair in $C$. The *dynamic time warping* (DTW) cost of a monotone assignment $C$ is

$$\mathsf{dtw}(T_1, T_2, C) = \sum_{(p,q) \in C} \|p - q\|^2$$

where $\|p - q\|$ is the Euclidean distance between $p$ and $q$. Define $\mathsf{dtw}(T_1, T_2) = \arg\min_C \mathsf{dtw}(T_1, T_2, C)$, where the minimum is taken over all monotone assignments.

(a) [10pts] Describe an $O(rs)$-time algorithm to compute $\mathsf{dtw}(T_1, T_2)$.

(b) [10pts] Suppose that $s$ is much larger than $r$, and we want to find a contiguous subsequence $R_2$ of $T_2$ such that $\mathsf{dtw}(T_1, R_2)$ is minimized among all contiguous subsequences of $T_2$. How would you adapt the algorithm in (a) to compute $\mathsf{dtw}(T_1, R_2)$ such that the asymptotic running time stays the same?