Due Date: October 28, 11:59pm

Problem 1: [10pts] Given points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ in the plane, we want to measure how closely the line $y = ax + b$ with $a, b \in \mathbb{R}$ fits the set of points.

(a) [5pts] The L_1 error of the line $y = ax + b$ is defined as follows:

$$\varepsilon_1(a, b) = \sum_{i=1}^{n} |y_i - ax_i - b|$$

Describe a linear program whose solution (a, b) describes the line with minimum L_1 error.

(b) [5pts] The L_∞ error of the line $y = ax + b$ is defined as follows:

$$\varepsilon_\infty(a, b) = \max_{i=1}^{n} |y_i - ax_i - b|$$

Describe a linear program whose solution (a, b) describes the line with minimum L_∞ error.

Problem 2: [10pts] Recall the LP formulation of the adword problem discussed in class:

$$\begin{aligned}
\max \quad & \sum_{\substack{1\le i\le n \\ 1\le j\le m}} b_{ij}x_{ij} \\
\text{subject to} \quad & \sum_{i=1}^{n} x_{ij} \le 1 \quad 1 \le j \le m \\
& \sum_{j=1}^{m} b_{ij}x_{ij} \le B_i \quad 1 \le i \le n \\
& x_{ij} \ge 0
\end{aligned}$$

Write the dual of this linear program.

Problem 3: [10pts]

(a) [5pts] Consider the maximum weighted matching problem in a weighted bipartite graph $G = (U \cup V, E)$, where $E \subseteq U \times V, w : E \to \mathbb{R}_{\ge 0}$. Consider its relaxed LP formulation:

$$\begin{aligned}
\max \quad & \sum_{(i,j)\in E} w(i,j)x_{ij} \\
\text{subject to} \quad & \sum_{j\in V} x_{ij} \le 1 \quad \forall i \in U \\
& \sum_{i\in U} x_{ij} \le 1 \quad \forall j \in V \\
& x_{ij} \ge 0
\end{aligned}$$

Show that there is an optimal solution that is integral, i.e., in which each $x_{ij} \in \{0, 1\}$.

(b) [5pts] Now dualize the linear program from part (a). What do the dual variables represent? What does the objective function represent?