Due Date: October 28, 11:59pm

Problem 1: [10pts] Given points \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\) in the plane, we want to measure how closely the line \(y = ax + b\) with \(a, b \in \mathbb{R}\) fits the set of points.

(a) [5pts] The \(L_1\) error of the line \(y = ax + b\) is defined as follows:

\[
\varepsilon_1(a, b) = \sum_{i=1}^{n} |y_i - ax_i - b|
\]

Describe a linear program whose solution \((a, b)\) describes the line with minimum \(L_1\) error.

(b) [5pts] The \(L_\infty\) error of the line \(y = ax + b\) is defined as follows:

\[
\varepsilon_\infty(a, b) = \max_{i=1}^{n} |y_i - ax_i - b|
\]

Describe a linear program whose solution \((a, b)\) describes the line with minimum \(L_\infty\) error.

Problem 2: [10pts] Recall the LP formulation of the adword problem discussed in class:

\[
\begin{align*}
\text{max} & \quad \sum_{1 \leq i \leq n} \sum_{1 \leq j \leq m} b_{ij} x_{ij} \\
\text{subject to} & \quad \sum_{j} x_{ij} \leq 1 \quad 1 \leq j \leq m \\
& \quad \sum_{i} b_{ij} x_{ij} \leq B_i \quad 1 \leq i \leq n \\
& \quad x_{ij} \geq 0
\end{align*}
\]

Write the dual of this linear program.

Problem 3: [10pts]

(a) [5pts] Consider the maximum weighted matching problem in a weighted bipartite graph \(G = (U \cup V, E)\), where \(E \subseteq U \times V, w : E \rightarrow \mathbb{R}_{\geq 0}\). Consider its relaxed LP formulation:

\[
\begin{align*}
\text{max} & \quad \sum_{(i,j) \in E} w(i, j) x_{ij} \\
\text{subject to} & \quad \sum_{j \in V} x_{ij} \leq 1 \quad \forall i \in U \\
& \quad \sum_{i \in U} x_{ij} \leq 1 \quad \forall j \in V \\
& \quad x_{ij} \geq 0
\end{align*}
\]

Show that there is an optimal solution that is integral, i.e., in which each \(x_{ij} \in \{0, 1\}\).

(b) [5pts] Now dualize the linear program from part (a). What do the dual variables represent? What does the objective function represent?