Due Date: November 4, 11:59pm

Problem 1: [10pts] Let $G = (V, E)$ be a graph with a weight function $w : E \to \mathbb{R}_{\geq 0}$. Consider the following linear program for minimum spanning trees in G:

$$
\begin{aligned}
\text{min} \quad & \sum_{e \in E} w(e) \cdot x_e \\
\text{subject to} \quad & \sum_{e \in E(S,\overline{S})}^{n} x_e \geq 1 \quad \text{for all } S \subseteq V, \overline{S} = V - S \\
& x_e \geq 0
\end{aligned}
$$

Given any $x \in \mathbb{R}^E$, a separation oracle determines whether x is a feasible solution to the linear program in polynomial time. If x is infeasible, it also returns a constraint not satisfied by x. Design a separation oracle for the exponential set of constraints, and analyze the running time.

Problem 2: [10pts] Solve the following linear proram using the simplex method, with the slack variables as the initial basis. In the first step, x_1 should be the variable entering the basis.

$$
\begin{aligned}
\text{max} \quad & -x_1 + x_2 \\
\text{subject to} \quad & x_1 - x_2 \leq 2 \\
& 2x_1 - x_2 \leq 6 \\
& -x_1 + x_2 \leq 4 \\
& x_1 \leq 6 \\
& x_2 \leq 8 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Draw the feasible region in 2D. Write the sequence of basis. Show the simplex tableau for each basis, and draw the sequence of vertices visited by the algorithm.