CPS 570: Artificial Intelligence

Homework 0: Search Space Sizes / Combinatorics (due September 2 11:59pm US Eastern)

Please read the rules for assignments on the course web page (http://www2.cs.duke.edu/courses/fall21/compsci570/). Please use Ed Discussion for questions (use private questions if your question is likely to reveal part of the answer to others). Use Gradescope to turn this in.

1. Consider the game of tic-tac-toe, where players alternately move. Player 1 (who moves first) puts Xs, and player 2 puts Os, on the 3×3 board. Normally in this game, we stop when someone has three in a row. However, now suppose that we always continue all the way until the board is full. So the players move in the sequence 1, 2, 1, 2, 1, 2, 1, 2, 1.
a. How many possible *final states* of the board are there? (Note: if one state can be obtained from another by rotating or flipping the board, this does **not** mean that these states are the same. But the *order* in which Os and Xs were put on the board does not matter, as long as the board looks the same at the end. I.e., if the board looks the same, it's the same state.)

b. How many different possibilities are there for the *full history of the game* until the end? I.e., how many different sequences of play until the end are there? If two sequences of play end up with the same final state, but the Xs and Os were put in in a different order, these are two different sequences. (Hint: One might represent a sequence by a final state of the board, plus, for every space on the board, the time—from 1 through 9—at which this space was filled.)

c. Now suppose there is a third player who puts As. Players move in the sequence 1, 2, 3, 1, 2, 3, 1, 2, 3. Answer question **a** again now that there is a third player. (Hint: consider the number of ways there are to first choose l elements out of a set of n elements, and to then choose i elements out of those l. The number of ways is

$$\binom{n}{l} \cdot \binom{l}{i} = \frac{n!}{l!(n-l)!} \cdot \frac{l!}{i!(l-i)!} = \frac{n!}{(n-l)!i!(l-i)!}$$

Intuitively, what we are doing is splitting n up into 3 sets, of sizes $n-l$, i, and $l-i$ respectively. Generally, the number of ways to split n elements into three sets of sizes i, j, k with $i+j+k=n$ is $\frac{n!}{i!j!k!}$.)

d. Answer question **b** again now that there is a third player.

2. Consider the "rooks problem" where we try to put 8 rooks on an 8x8 chess board in such a way that no pair attacks each other. (A rook can move either horizontally or vertically, any distance, in one move.)
a. How many different solutions are there to this?

b. Suppose we place the rooks on the board one by one, and we care about the order in which we put them on the board. We still cannot place them in ways that attack each other. How many different full sequences of placing the rooks (ending in one of the solutions from **a**) are there? As in **1b**, two sequences that end up in the same final state but in which the rooks were added in a different order are still different. Find two different ways to count the number of sequences and check that they give the same answer.