COMPSCI 570: Artificial Intelligence

Homework 3: First-order logic (due October 19, 11:59pm)

Please read the rules for assignments on the course web page (http://www2.cs.duke.edu/courses/fall21/compsci570/) Please use Ed Discussion for questions (use private questions if your question is likely to reveal part of the answer to others). Use Gradescope to turn this in.

If you want to use LaTeX that is great. If you write by hand and scan, please make sure that your handwriting is clear; illegible/ambiguous handwriting will receive no points. Please make sure that your answers are clear and be careful with parentheses and symbols.

In your gradescope submission, please submit each section as a separate page, making it a total of four pages. It is acceptable to have multiple pages for a problem.

1 Convert the following English sentences to first-order logic **(4 * 5 = 20 points)**

1. Every thing that is an enemy of some thing that is an enemy of me is a friend of me ("the enemy of my enemy is my friend").
2. Every person who is smart and studies hard will get a higher score than every person who is not smart and does not study hard.
3. A silver medal is worth more than a bronze medal, if they are medals in the same event.
4. Every thing that walks like a duck and talks like a duck is either a duck or a human imitating a duck.

2 Choose all correct first-order logic options for given English sentences. Some questions have multiple correct options; choose all options that are correct **(4 * 5 = 20 points)**

1. No one who cheats wins.
   (a) $\neg\exists x\ (\text{cheats}(x) \wedge \text{wins}(x))$
   (b) $\forall x\ (\neg\text{cheats}(x) \Rightarrow \text{wins}(x))$
   (c) $\neg\exists x\ (\text{cheats}(x) \Rightarrow \text{wins}(x))$
   (d) $\forall x\ (\text{cheats}(x) \Rightarrow \neg\text{wins}(x))$

2. If anyone is noisy, everyone is annoyed.

   (a) $\forall y\ ((\forall x\ \text{noisy}(x)) \Rightarrow \text{annoyed}(y))$
   (b) $\forall x \forall y\ (\text{noisy}(x) \Rightarrow \text{annoyed}(y))$
   (c) $(\exists x\ \text{noisy}(x)) \Rightarrow \forall y\ \text{annoyed}(y)$
   (d) $\forall y \exists x\ (\text{noisy}(x) \Rightarrow \text{annoyed}(y))$

3. Mary does not hate anyone.

   (a) $\neg\exists x$ hate(Mary,x)
   (b) $\exists x\ \neg$hate(Mary,x)
   (c) $\forall x\ \neg$hate(Mary,x)
   (d) $\neg\forall x$ hate(Mary,x)

4. Some boys in the class are taller than all the girls.

   (a) $(\exists x)\ (\text{boy}(x) \Rightarrow (\forall y)\ (\text{girl}(y) \wedge \text{taller}(x, y)))$
   (b) $(\exists x)\ (\text{boy}(x) \wedge (\forall y)\ (\text{girl}(y) \wedge \text{taller}(x, y)))$
   (c) $(\exists x)\ (\text{boy}(x) \Rightarrow (\forall y)\ (\text{girl}(y) \Rightarrow \text{taller}(x, y)))$
   (d) $(\exists x)\ (\text{boy}(x) \wedge (\forall y)\ (\text{girl}(y) \Rightarrow \text{taller}(x, y)))$

3 Apply resolution to obtain the most general conclusion possible (20 points)

Write the conclusion both in first-order logic and in English.

- $\forall x, y$: LovesTheCombinationOf(John, x, y) $\vee$ MakesSick(x,John) $\vee$ RuinsTasteOf(y, x)
- $\forall v, w$: $\neg$ LovesTheCombinationOf(v, Rice, w) $\vee$ Flavorful(w)

For brevity, use: LTCO = LovesTheCombinationOf, MS = MakesSick, RTO = RuinsTasteOf, F = Flavorful Hint: Set up the knowledge base. Use substitutions and resolution to arrive at the conclusion.

4 Enemies and Friends (40 points)

Suppose you know the following.

1. For any x, any enemy of any enemy of x is a friend of x. (stated more naturally in English, in 1.1)
2. If x is an enemy of y, then y is an enemy of x.
   $\forall x, y$: enemyOf$(x, y) \Rightarrow$ enemyOf(y, x)
3. Every x has at least two enemies.
   $\forall x : \exists y, z : \neg(y = z) \wedge$ enemyOf$(x, y) \wedge$ enemyOf(x, z)

Use a theorem prover (such as the one on the website we used in class) to formally prove that Alice has at least one friend that is not equal to herself. This, of course, requires you to translate this last statement into first-order logic as well. Include both your input and the output you got. You do not need to write anything about the output that you got, but you are encouraged to check it out, also to make sure you didn't make a mistake in the input.