

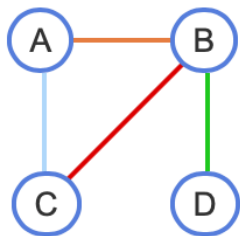
# CompSci 201, L23: Iterative DFS BFS

# Logistics, Coming up

- APT10 (greedy) due today, Wed., 11/16
- APT Quiz 2: 2 hours, 3 problems
  - covers APT6-10, linked list and tree problems guaranteed
  - Release: This Thursday 11/17
  - Complete by: Monday 11/21
  - Quiz, not a hw, no late period
- Project 6: Route releasing this week, due Monday 12/5

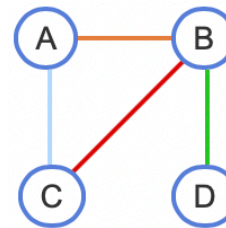
# General data structures for graphs: Not necessarily a grid

## Adjacency List



Vertices	Adjacent vertices (edges)		
A	B	C	
B	A	C	D
C	A	B	
D	B		

## Adjacency Matrix

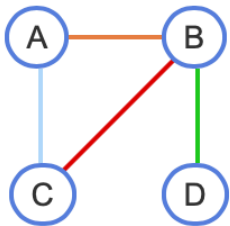


	A	B	C	D
A		1	1	
B	1		1	1
C	1	1		
D		1		

Zybook chapter 23

# Efficient Adjacency “List” Using Double Hashing

- `HashMap<Vertex, HashSet<Vertex>> aList`
  - Vertex type can be Integer, char, String, custom object, ..., needs to have good `hashCode()` and `equals()`.



Vertices	Adjacent vertices (edges)
A	B C
B	A C D
C	A B
D	B

- `aList.put('A', new HashSet())`
- `aList.get('A').add('B')`
- `aList.get('A').add('C')`
- ...

$O(1)$  time to check if nodes are connected or get the neighbors of a node (assuming good `hashCode`)

# Graph Search Data Structures

- Have an adjacency list for the graph
- Keep track of visited nodes in a set
- Keep track of the *previous* node: During search, how did I get to this node?

```
9  public class DFS {  
10      public static Map<Character, Set<Character>> aList;  
11      public static Set<Character> visited;  
12      public static Map<Character, Character> previous;
```

- Example has Character nodes, could be any label for the nodes.
- Storing as instance variables, accessible in methods.

# Iterative Depth-First Search (DFS)

# Initializing Iterative DFS

- **Stack** stores nodes we have *visited/discovered*, but not explored from yet.
- Explore from one *current* node at a time.

```
14     public static void dfs(char start) {  
15         Stack<Character> toExplore = new Stack<>();  
16         char current = start;  
17         toExplore.add(current);  
18         visited.add(current);
```

- Stack is LIFO (last-in first-out), so we always explore from the *last node we discovered*, **depth-first!**

# Iterative DFS Loop

While there are nodes we have not explored from...

Explore from the most recently discovered node...

Look at all neighbors of current node...

If we haven't seen them before...

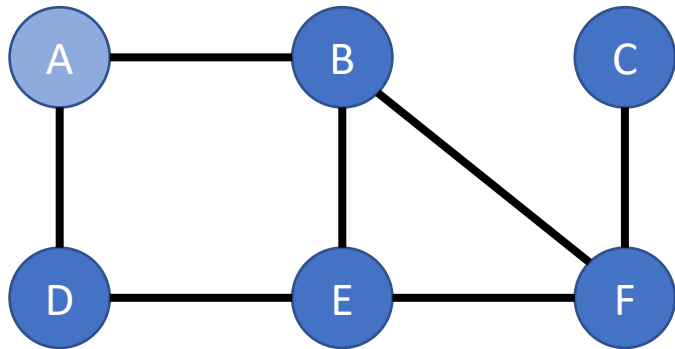
Then:  
1. note how we got here  
2. Note we have seen  
3. Mark to explore later

```
20 while (!toExplore.isEmpty()) {  
21     current = toExplore.pop();  
22     for (char neighbor : aList.get(current)) {  
23         if (!visited.contains(neighbor)) {  
24             previous.put(neighbor, current);  
25             visited.add(neighbor);  
26             toExplore.push(neighbor);  
27         }  
28     }  
29 }
```



# Initialize search at A

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

previous (map)

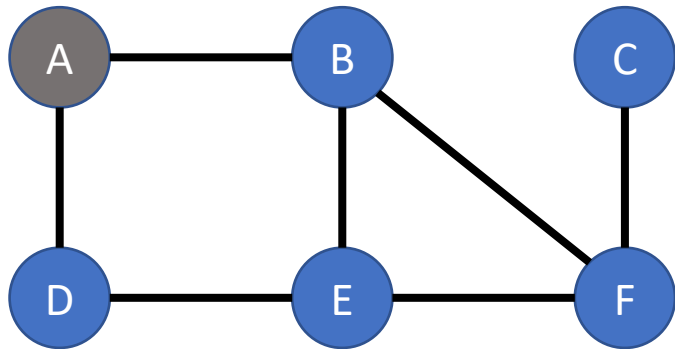
Visited (set)

A

{A}

# Pop A off the stack

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

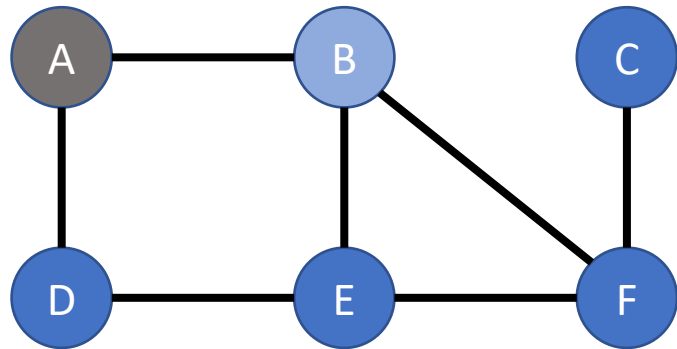
previous (map)

Visited (set)

{A}

# Find B from A

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

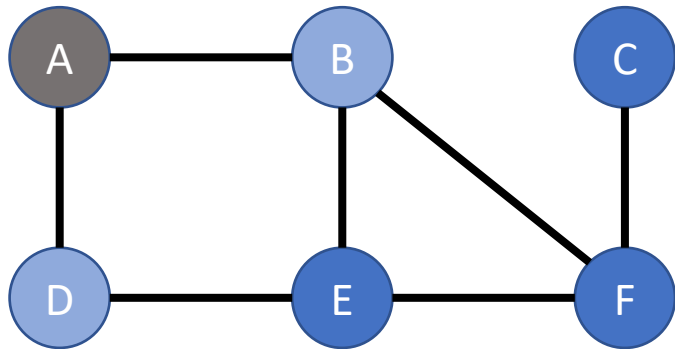
B

B ← A

{A, B}

# Find D from A

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

D

B ← A

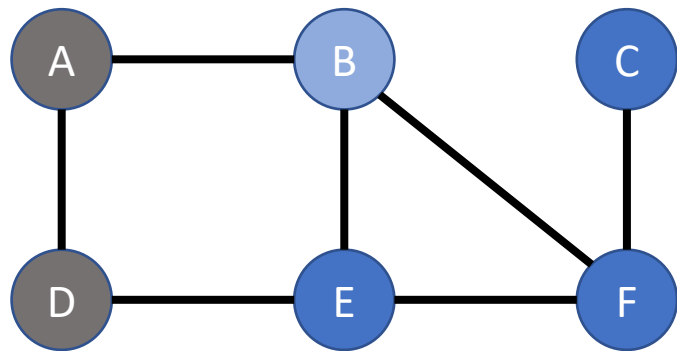
{A, B, D}

B

D ← A

# Pop D off the stack

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

B

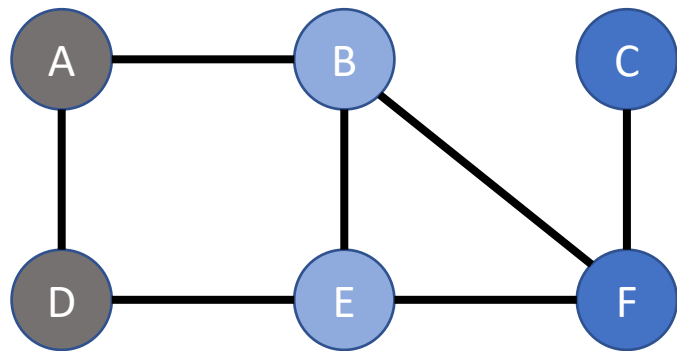
B  $\leftarrow$  A

{A, B, D}

D  $\leftarrow$  A

# Find E from D

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

E

B ← A

{A, B, D, E}

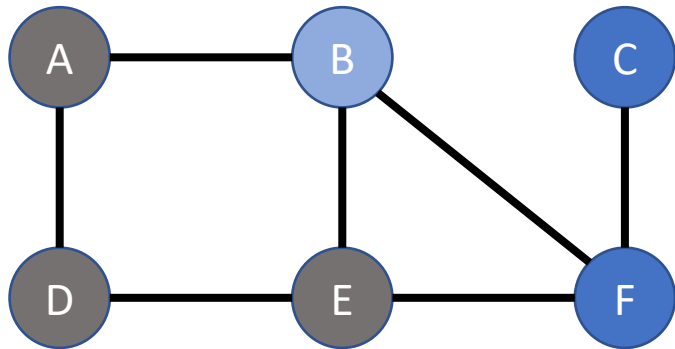
B

D ← A

E ← D

# Pop E off the stack

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

B

previous (map)

B  $\leftarrow$  A

D  $\leftarrow$  A

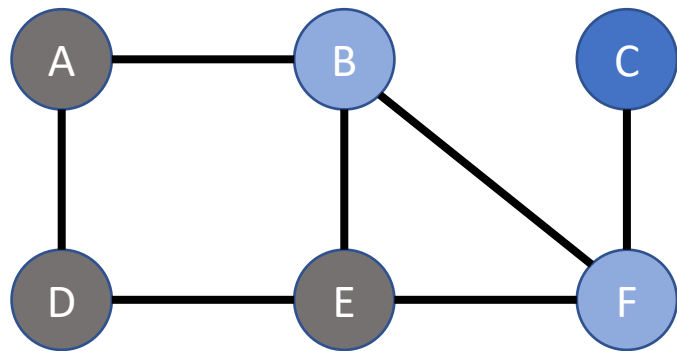
E  $\leftarrow$  D

Visited (set)

{A, B, D, E}

# Find F from E

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

F

B  $\leftarrow$  A

{A, B, D, E, F}

B

D  $\leftarrow$  A

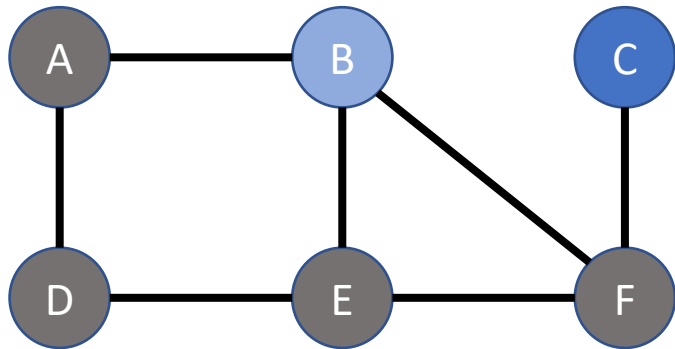
E  $\leftarrow$  D

F  $\leftarrow$  E



# Pop F off the stack

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

B

B  $\leftarrow$  A

{A, B, D, E, F}

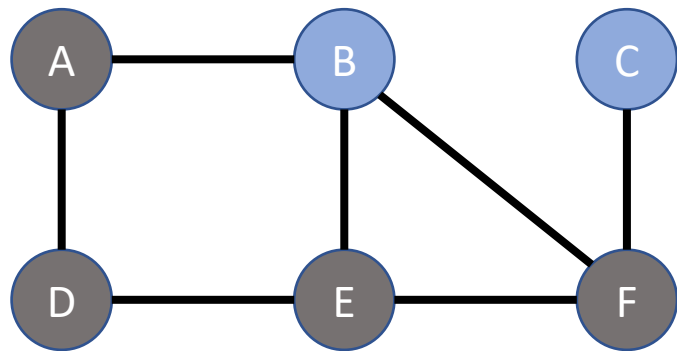
D  $\leftarrow$  A

E  $\leftarrow$  D

F  $\leftarrow$  E

# Find C from F

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

C

B

B  $\leftarrow$  A

D  $\leftarrow$  A

E  $\leftarrow$  D

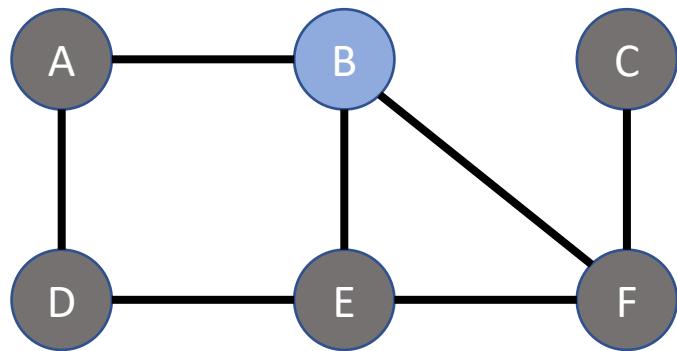
F  $\leftarrow$  E

C  $\leftarrow$  F

{A, B, D, E, F, C}

# Pop C off the stack

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

B

B  $\leftarrow$  A

{A, B, D, E, F, C}

D  $\leftarrow$  A

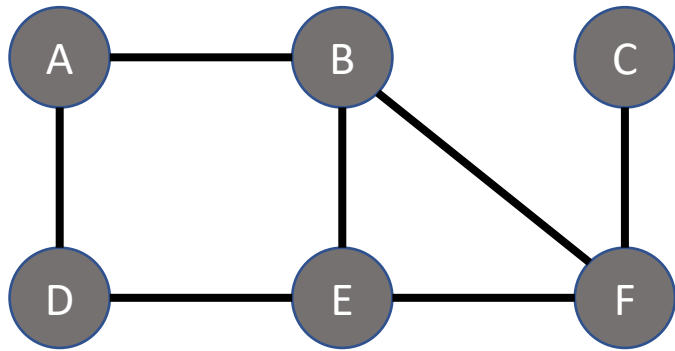
E  $\leftarrow$  D

F  $\leftarrow$  E

C  $\leftarrow$  F

# Pop B off the stack

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

B ← A

D ← A

E ← D

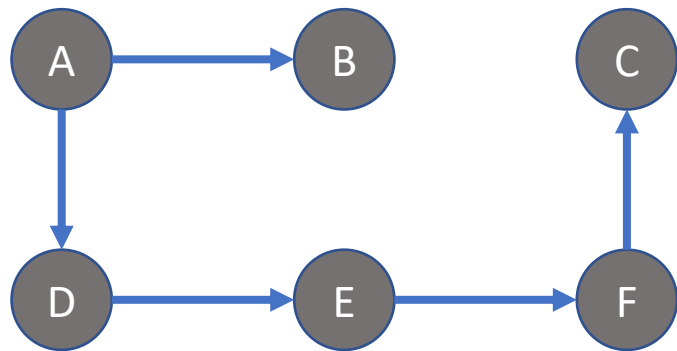
F ← E

C ← F

{A, B, D, E, F, C}

# DFS Search Tree

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

Can find paths from A to X by following previous backwards from X

B <- A

D <- A

E <- D

F <- E

C <- F

{A, B, D, E, F, C}

Path from A to C:  
C <- F <- E <- D <- A

# WOTO

Go to [duke.is/m467a](https://duke.is/m467a)

Not graded for correctness,  
just participation.

Try to answer *without* looking  
back at slides and notes.

But do talk to your neighbors!



# DFS Complexity?

While loop over all nodes (N), potentially?

Loop over edges (M)

Seems like  $O(NM)$ ,  
but...

```
20 while (!toExplore.isEmpty()) {
21     current = toExplore.pop();
22     for (char neighbor : aList.get(current)) {
23         if (!visited.contains(neighbor)) {
24             previous.put(neighbor, current);
25             visited.add(neighbor);
26             toExplore.push(neighbor);
27         }
28     }
29 }
```

# DFS Complexity?

```
20 while (!toExplore.isEmpty()) {  
21     current = toExplore.pop();  
22     for (char neighbor : aList.get(current)) {  
23         if (!visited.contains(neighbor)) {  
24             previous.put(neighbor, current);  
25             visited.add(neighbor);  
26             toExplore.push(neighbor);  
27         }  
28     }  
29 }
```

Loop over edges **adjacent**  
to current node

- Pop each of  $N$  nodes *at most once*.
- Loop over neighbors of each node *exactly once*, considers each edge twice.
- $N+2M$  is  $O(N+M)$ .



# Iterative Breadth-First Search (BFS)

# Queue: A FIFO List

- Both add and remove are  $O(1)$ 
  - Add at end of LinkedList
  - Remove from front of LinkedList

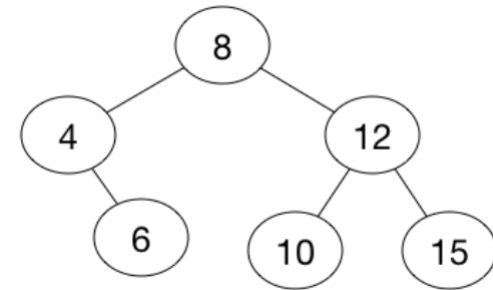
LinkedList implements the Queue interface.

```
5 public static void gdemo() {  
6     String[] strs = {"compsci", "is", "wonderful"};  
7     Queue<String> q = new LinkedList<>();  
8     for(String s : strs) {  
9         q.add(s);  
10    }  
11    while (! q.isEmpty()) {  
12        System.out.println(q.remove());  
13    }  
14 }
```

compsci  
is  
wonderful

# levelOrder Tree Traversal with a queue

```
public static void levelOrder(TreeNode tree) {  
    Queue<TreeNode> queue = new LinkedList<>();  
    queue.add(tree);  
    while (!queue.isEmpty()) {  
        TreeNode current = queue.remove();  
        if (current != null) {  
            System.out.println(current.info);  
            queue.add(current.left);  
            queue.add(current.right);  
        }  
    }  
}
```



8  
4  
12  
6  
10  
15

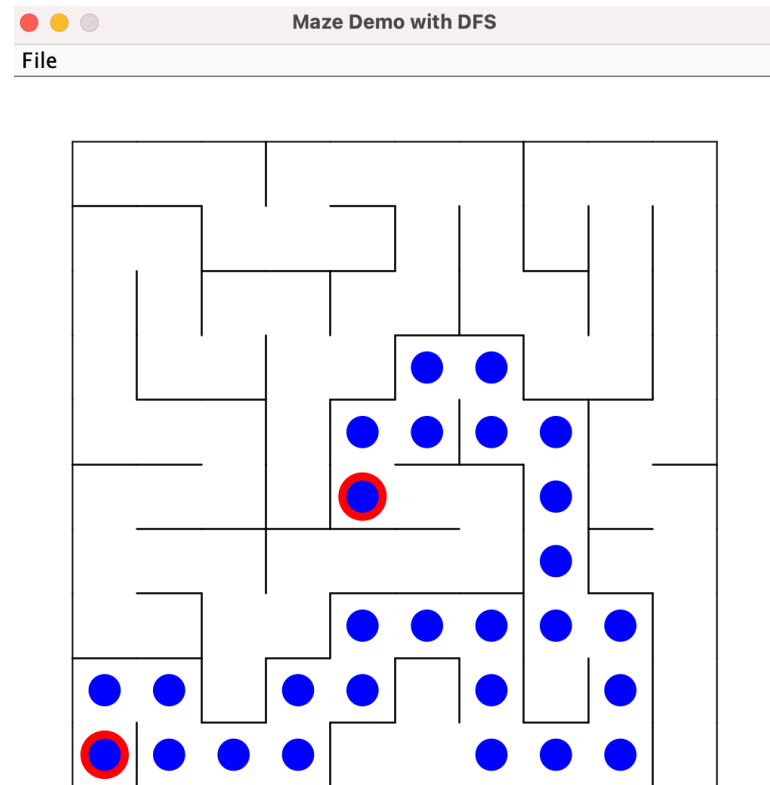
Use a queue to keep track of nodes  
First in first out, nodes visited in level order

# Depth First Search for Solving Maze

Always explore (recurse on) a new (unvisited) adjacent vertex if possible.

If impossible, **backtrack** to the most recent vertex adjacent to an unvisited vertex and continue.

[coursework.cs.duke.edu/cs-201-fall-22/maze-demo](https://coursework.cs.duke.edu/cs-201-fall-22/maze-demo)

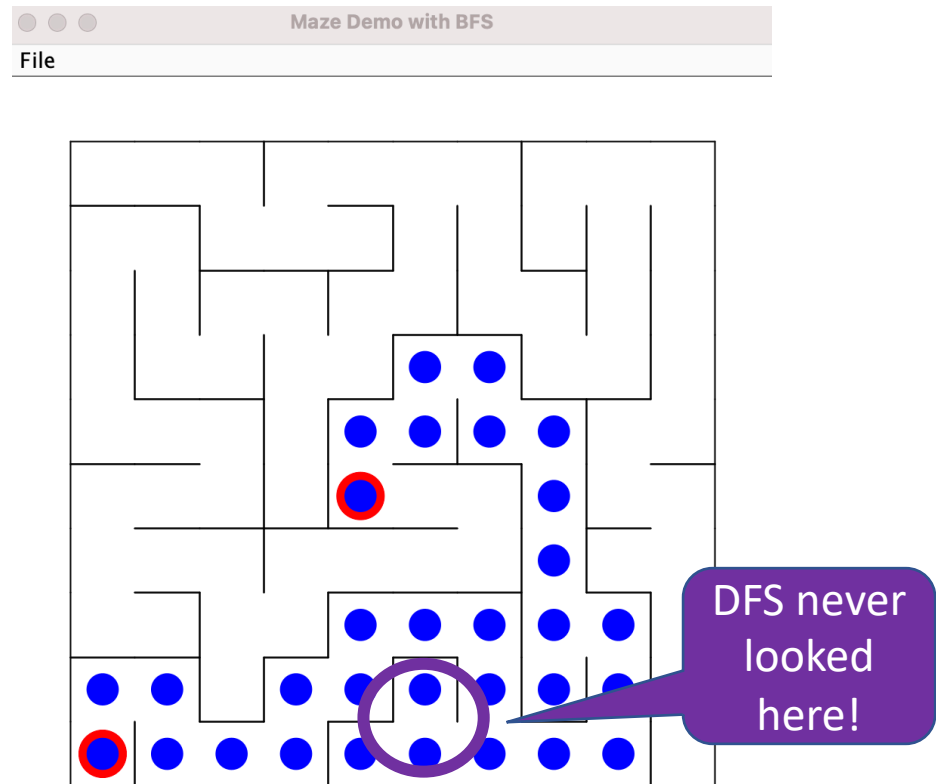


# Breadth First Search for Solving Maze

[coursework.cs.duke.edu/cs-201-fall-22/maze-demo](https://coursework.cs.duke.edu/cs-201-fall-22/maze-demo)

Explore *all* your neighbors (adjacent vertices) before you visit any of your neighbors' neighbors.

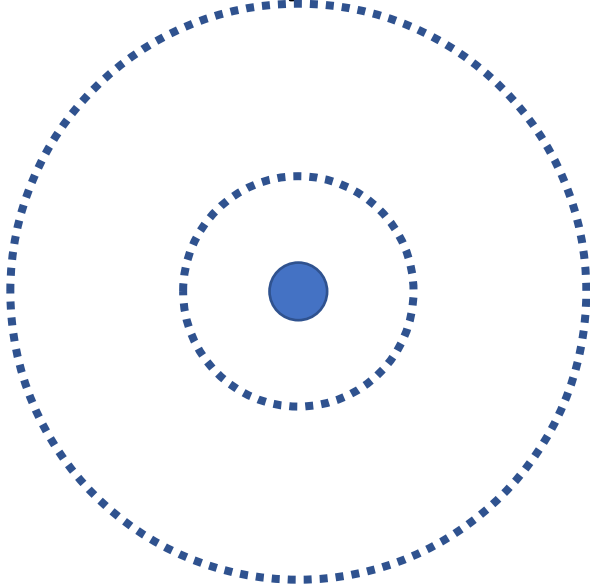
Looking for the shortest path/solution.



# Queue = BFS, Stack = DFS

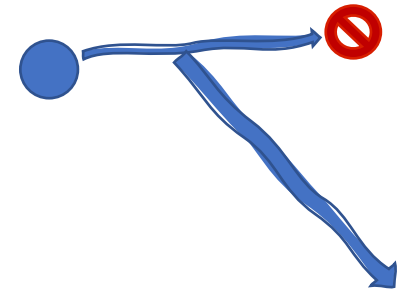
## BFS: FIFO Exploration

search all locations one-away from start, then two-away, ...



## DFS: LIFO Exploration

Search path as far as possible, backtrack if need to another branch...



# Initializing Iterative BFS

- **Queue** stores nodes we have *visited/discovered*, but not explored from yet.
- Explore from one *current* node at a time.

```
32     public static void bfs(char start) {  
33         Queue<Character> toExplore = new LinkedList<>();  
34         char current = start;  
35         visited.add(current);  
36         toExplore.add(current);
```

- Queue is FIFO(first-in first-out), so we always explore from the *first/closest (unvisited) node we discovered*, **breadth-first!**

# Iterative BFS Loop

While there are nodes we have not explored from...

```
38 while (!toExplore.isEmpty()) {  
39     current = toExplore.remove();  
40     for (char neighbor : aList.get(current)) {  
41         if (!visited.contains(neighbor)) {  
42             previous.put(neighbor, current);  
43             visited.add(neighbor);  
44             toExplore.add(neighbor);  
45         }  
46     }  
47 }
```

Explore from the **closest** discovered node...

Look at all neighbors of current node...

If we haven't seen them before...

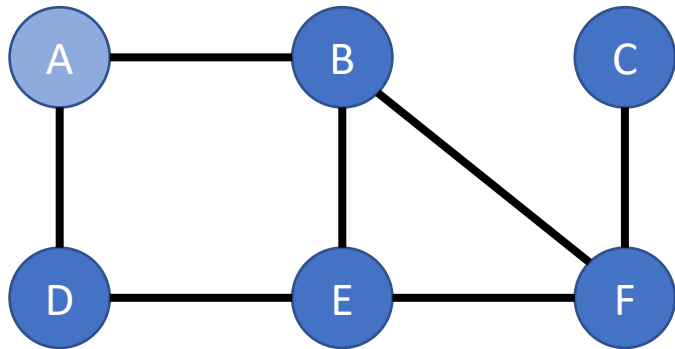
Then:

1. note how we got here
2. Note we have seen
3. Mark to explore later



# Initialize search at A

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

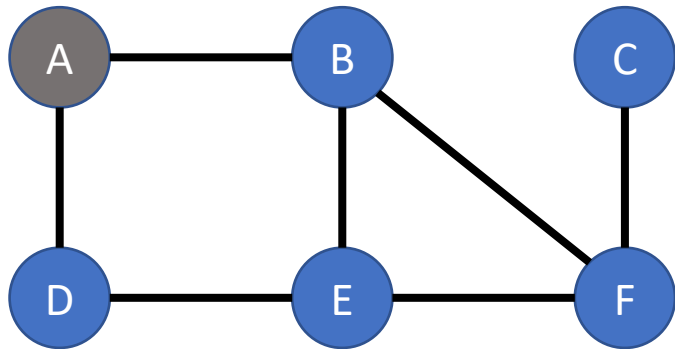
toExplore (queue)    previous (map)    Visited (set)

A

{A}

# Remove A from the queue

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

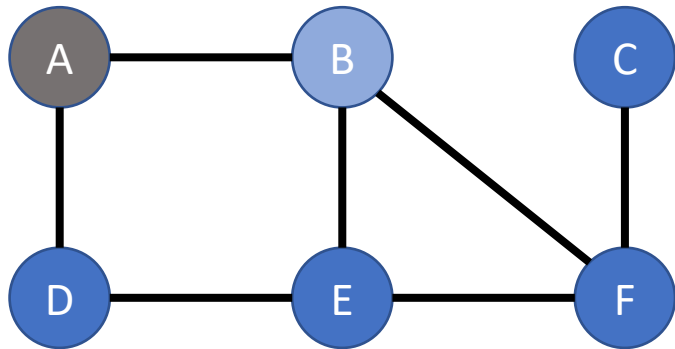
F=[B, C, E]

toExplore (queue)    previous (map)    Visited (set)

{A}

# Find B from A

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

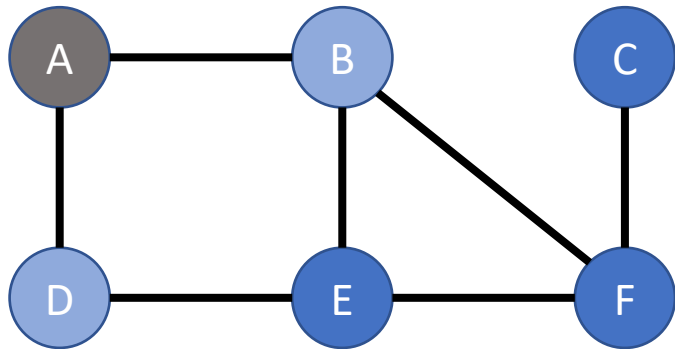
F=[B, C, E]

toExplore (queue)	previous (map)	Visited (set)
-------------------	----------------	---------------

B	B ← A	{A, B}
---	-------	--------

# Find D from A

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

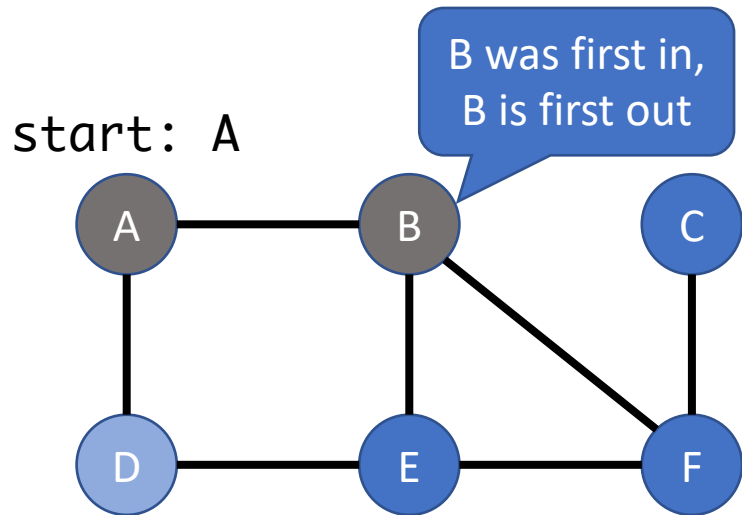
F=[B, C, E]

toExplore (queue)	previous (map)	Visited (set)
-------------------	----------------	---------------

B	B ← A	{A, B, D}
D	D ← A	

Note the difference,  
add to end of queue!

# Remove B from queue



## Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

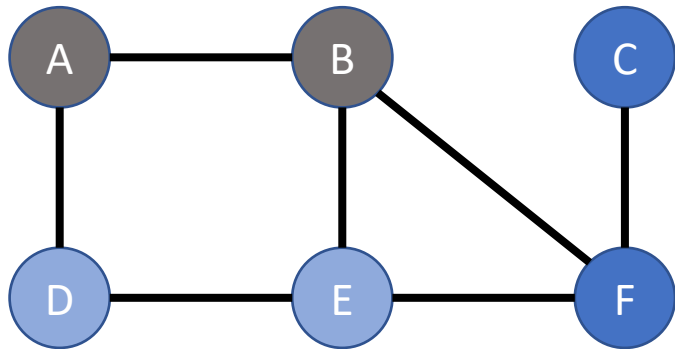
F=[B, C, E]

toExplore (queue)	previous (map)	Visited (set)
-------------------	----------------	---------------

D	B ← A	{A, B, D}
	D ← A	

# Find E from B

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (queue)

previous (map)

Visited (set)

D

B  $\leftarrow$  A

{A, B, D, E}

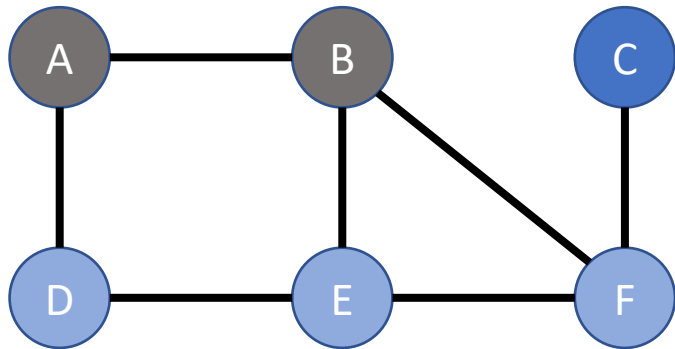
E

D  $\leftarrow$  A

E  $\leftarrow$  B

# Find F from B

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (queue)

previous (map)

Visited (set)

D

B  $\leftarrow$  A

{A, B, D, E, F}

E

D  $\leftarrow$  A

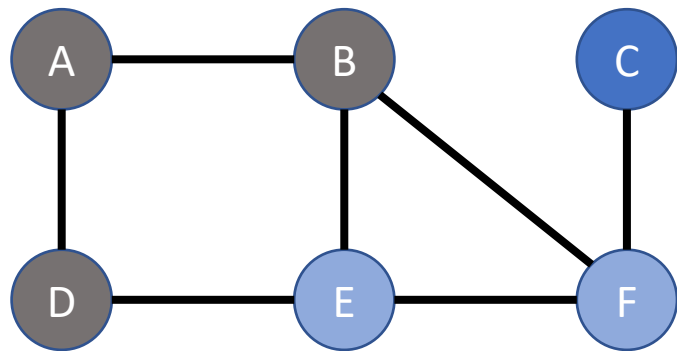
F

E  $\leftarrow$  B

F  $\leftarrow$  B

# Remove D from queue

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (queue)

previous (map)

Visited (set)

E

B  $\leftarrow$  A

{A, B, D, E, F}

F

D  $\leftarrow$  A

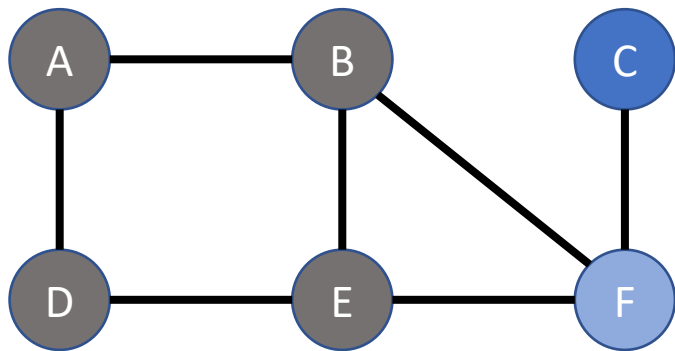
E  $\leftarrow$  B

F  $\leftarrow$  B



# Remove E from queue

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (queue)

previous (map)

Visited (set)

F

B  $\leftarrow$  A

{A, B, D, E, F}

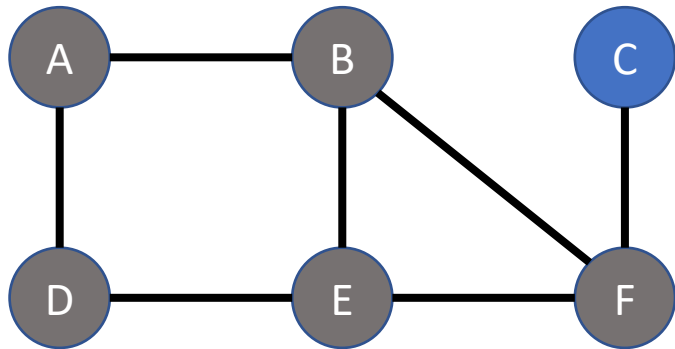
D  $\leftarrow$  A

E  $\leftarrow$  B

F  $\leftarrow$  B

# Remove F from queue

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (queue)

previous (map)

Visited (set)

B  $\leftarrow$  A

D  $\leftarrow$  A

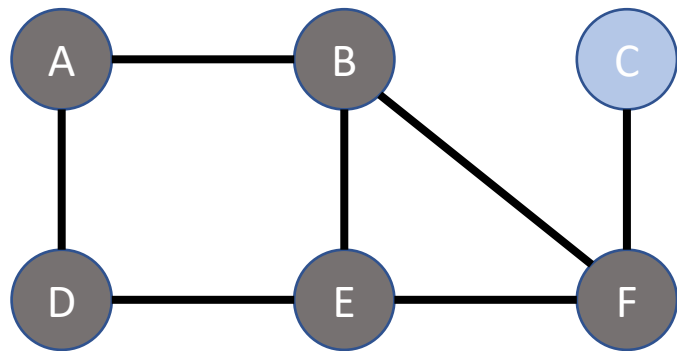
E  $\leftarrow$  B

F  $\leftarrow$  B

{A, B, D, E, F}

# Find C from F

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (queue)

previous (map)

Visited (set)

C

B  $\leftarrow$  A

{A, B, D, E, F, C}

D  $\leftarrow$  A

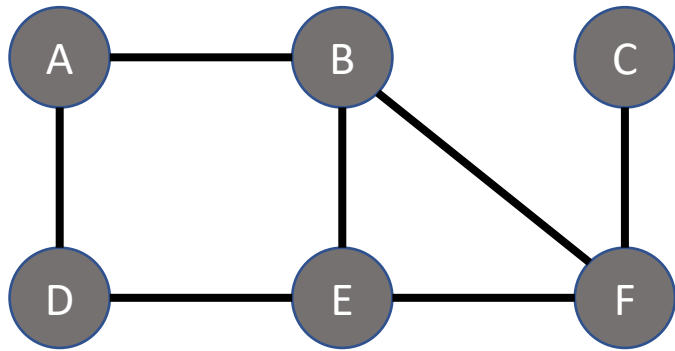
E  $\leftarrow$  B

F  $\leftarrow$  B

C  $\leftarrow$  F

# Remove C from queue

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (queue)

previous (map)

Visited (set)

B  $\leftarrow$  A

D  $\leftarrow$  A

E  $\leftarrow$  B

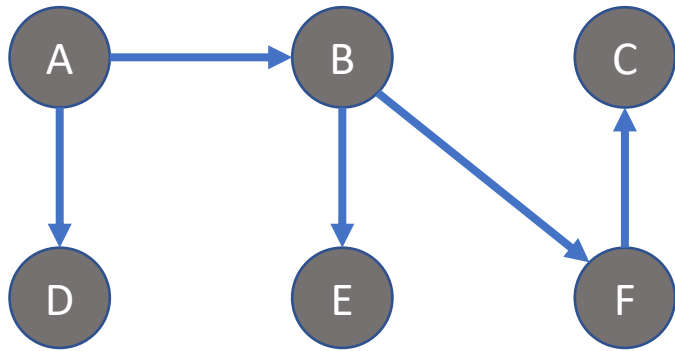
F  $\leftarrow$  B

C  $\leftarrow$  F

{A, B, D, E, F, C}

# BFS Search Tree

start: A



Adjacency List:

A=[B, D]

B=[A, E, F]

C=[F]

D=[A, E]

E=[B, D, F]

F=[B, C, E]

toExplore (queue)

previous (map)

Visited (set)

B ← A

D ← A

E ← B

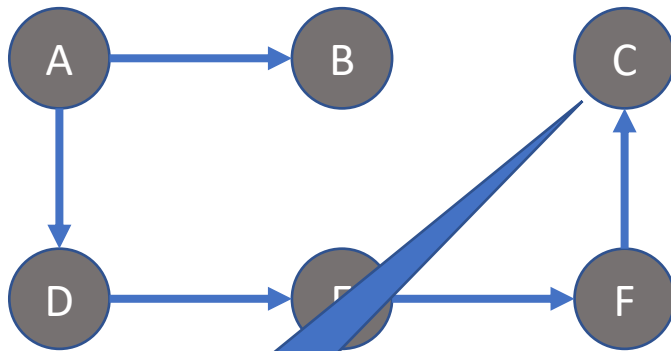
F ← B

C ← F

{A, B, D, E, F, C}

# Comparing DFS and BFS Search Trees

start: A

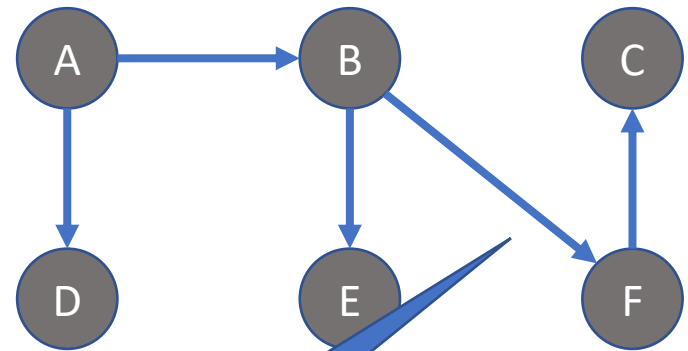


Length 4 path  
from A to C

previous (map)

```
B <- A
D <- A
E <- D
F <- E
C <- F
```

start: A



Length 3 path  
from A to C,  
shorter!

previous (map)

```
B <- A
D <- A
E <- B
F <- B
C <- F
```

# Pathfinding Properties

- DFS and BFS **both** find valid paths to *all* nodes reachable from the start.
  - Can return early if you only want to find a path to a specific target node
- BFS finds the *shortest path* to every reachable node, DFS does *not* guarantee this.

# WOTO

Go to [duke.is/wjrfp](https://duke.is/wjrfp)

Not graded for correctness,  
just participation.

Try to answer *without* looking  
back at slides and notes.

But do talk to your neighbors!

