

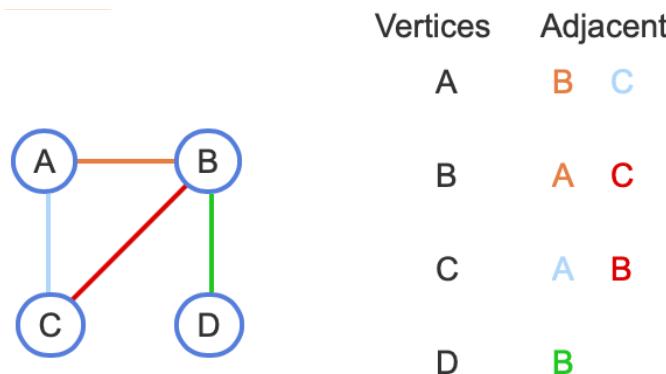
CompSci 201, L23: Iterative DFS BFS

Logistics, Coming up

- APT10 (greedy) due today, Wed., 11/16
- APT Quiz 2: 2 hours, 3 problems
 - covers APT6-10, linked list and tree problems guaranteed
 - Release: This Thursday 11/17
 - Complete by: Monday 11/21
 - Quiz, not a hw, no late period
- Project 6: Route releasing this week, due Monday 12/5

General data structures for graphs: Not necessarily a grid

Adjacency List



Adjacency Matrix

	A	B	C	D
A	1	1	0	0
B	0	1	1	1
C	1	0	1	0
D	0	1	0	1

Zybook chapter 23

Efficient Adjacency “List” Using Double Hashing

- `HashMap<Vertex, HashSet<Vertex>> aList`
 - Vertex type can be Integer, char, String, custom object, ..., needs to have good `hashCode()` and `equals()`.



- `aList.put('A', new HashSet())`
- `aList.get('A').add('B')`
- `aList.get('A').add('C')`
- ...

$O(1)$ time to check if nodes are connected or get the neighbors of a node (assuming good `hashCode`)

Graph Search Data Structures

- Have an adjacency list for the graph
- Keep track of visited nodes in a set
- Keep track of the *previous* node: During search, how did I get to this node?

```
9  public class DFS {  
10     public static Map<Character, Set<Character>> aList;  
11     public static Set<Character> visited;  
12     public static Map<Character, Character> previous;
```

- Example has Character nodes, could be any label for the nodes.
- Storing as instance variables, accessible in methods.

Iterative Depth-First Search (DFS)

Initializing Iterative DFS

- **Stack** stores nodes we have *visited/discovered*, but not explored from yet.
- Explore from one *current* node at a time.

```
14     public static void dfs(char start) {  
15         Stack<Character> toExplore = new Stack<>();  
16         char current = start;  
17         toExplore.add(current);  
18         visited.add(current);
```

- Stack is LIFO (last-in first-out), so we always explore from the *last node we discovered, depth-first!*

Iterative DFS Loop

While there are nodes we have not explored from...

Explore from the most recently discovered node...

```
20  while (!toExplore.isEmpty()) {  
21      current = toExplore.pop();  
22      for (char neighbor : aList.get(current)) {  
23          if (!visited.contains(neighbor)) {  
24              previous.put(neighbor, current);  
25              visited.add(neighbor);  
26              toExplore.push(neighbor);  
27      }  
28  }  
29 }
```

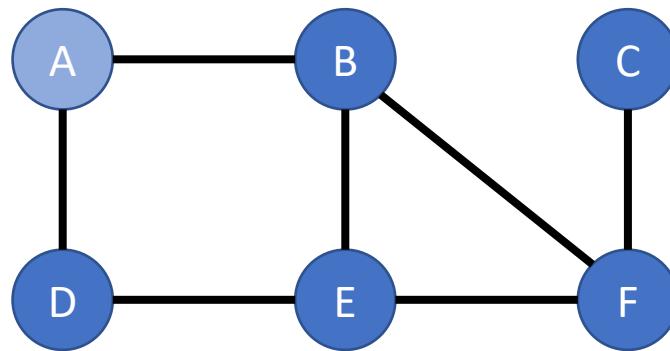
Look at all neighbors of current node...

If we haven't seen them before...

Then:
1. note how we got here
2. Note we have seen
3. Mark to explore later

Initialize search at A

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

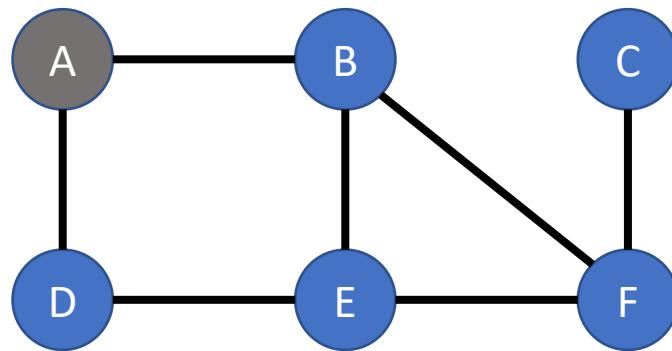
toExplore (stack) **previous (map)** **Visited (set)**

A

{A}

Pop A off the stack

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (stack)

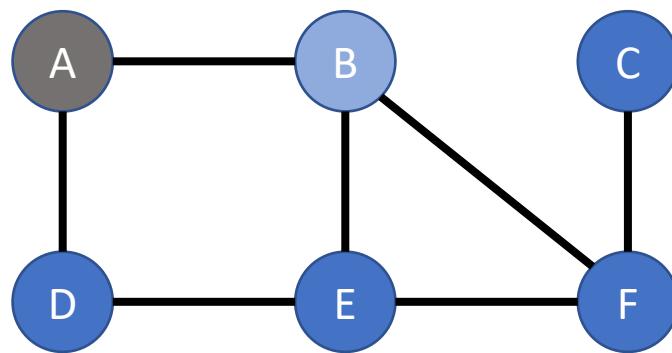
previous (map)

Visited (set)

{A}

Find B from A

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (stack)

B

previous (map)

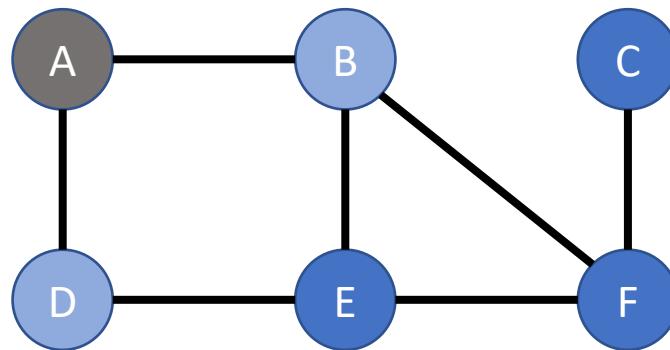
B <- A

Visited (set)

{A, B}

Find D from A

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (stack)

D
B

previous (map)

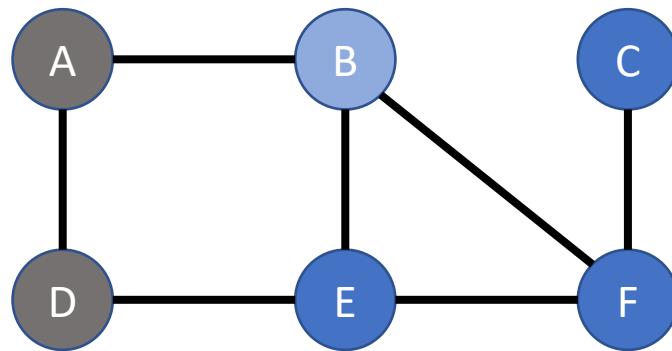
B <- A
D <- A

Visited (set)

{A, B, D}

Pop D off the stack

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (stack)

B

previous (map)

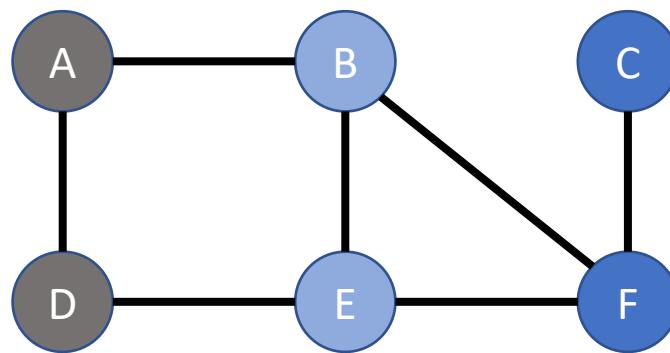
B <- A
D <- A

Visited (set)

{A, B, D}

Find E from D

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (stack)

E
B

previous (map)

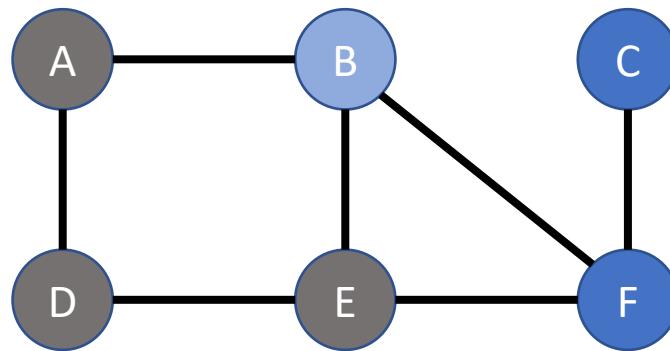
B <- A
D <- A
E <- D

Visited (set)

{A, B, D, E}

Pop E off the stack

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (stack)

B

previous (map)

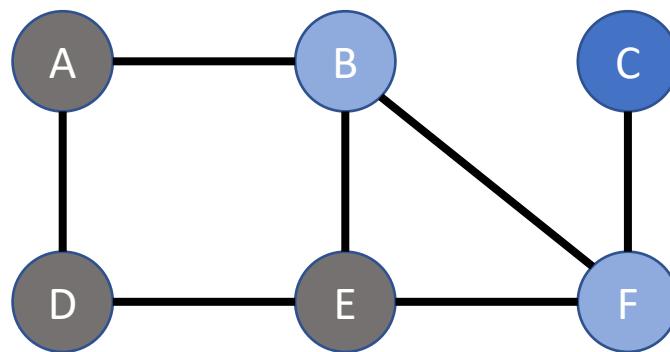
B <- A
D <- A
E <- D

Visited (set)

{A, B, D, E}

Find F from E

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (stack)

F
B

previous (map)

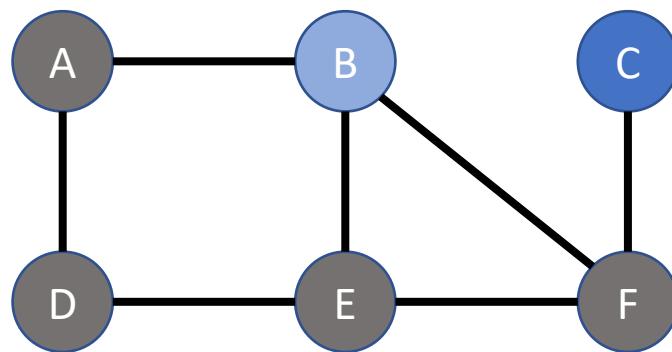
B <- A
D <- A
E <- D
F <- E

Visited (set)

{A, B, D, E, F}

Pop F off the stack

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (stack)

B

previous (map)

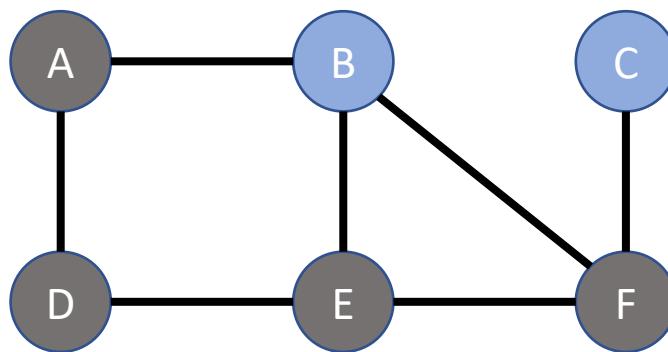
B <- A
D <- A
E <- D
F <- E

Visited (set)

{A, B, D, E, F}

Find C from F

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

C

B <- A

{A, B, D, E, F, C}

B

D <- A

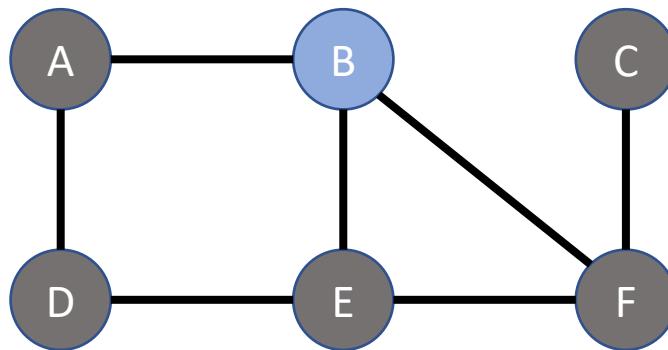
E <- D

F <- E

C <- F

Pop C off the stack

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (stack)

B

previous (map)

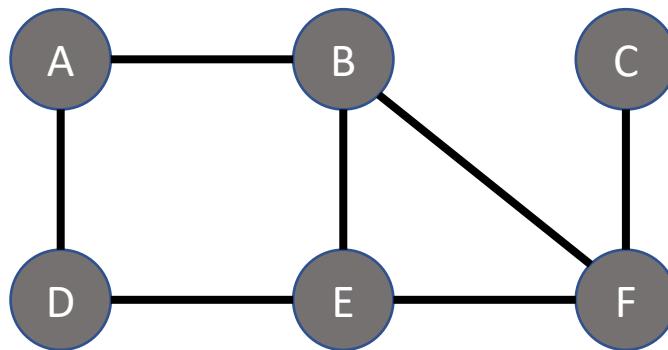
B <- A
D <- A
E <- D
F <- E
C <- F

Visited (set)

{A, B, D, E, F, C}

Pop B off the stack

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

B <- A

{A, B, D, E, F, C}

D <- A

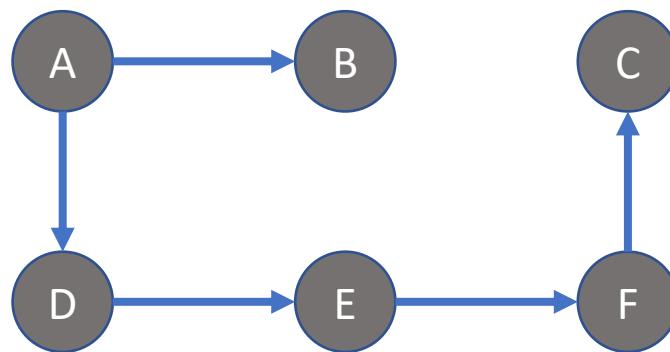
E <- D

F <- E

C <- F

DFS Search Tree

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (stack)

previous (map)

Visited (set)

Can find paths from A to X by following previous backwards from X

B <- A
D <- A
E <- D
F <- E
C <- F

{A, B, D, E, F, C}

Path from A to C:
C <- F <- E <- D <- A

WOTO

Go to duke.is/m467a

Not graded for correctness,
just participation.

Try to answer *without* looking
back at slides and notes.

But do talk to your neighbors!



DFS Complexity?

```
20 while (!toExplore.isEmpty()) {  
21     current = toExplore.pop();  
22     for (char neighbor : aList.get(current)) {  
23         if (!visited.contains(neighbor)) {  
24             previous.put(neighbor, current);  
25             visited.add(neighbor);  
26             toExplore.push(neighbor);  
27         }  
28     }  
29 }
```

While loop over all nodes (N), potentially?

Loop over edges (M)

Seems like $O(NM)$, but...

DFS Complexity?

```
20  while (!toExplore.isEmpty()) {           Loop over edges adjacent
21      current = toExplore.pop();           to current node
22      for (char neighbor : aList.get(current)) {
23          if (!visited.contains(neighbor)) {
24              previous.put(neighbor, current);
25              visited.add(neighbor);
26              toExplore.push(neighbor);
27      }
28  }
29 }
```

- Pop each of N nodes *at most once*.
- Loop over neighbors of each node *exactly once*, considers each edge twice.
- $N+2M$ is $O(N+M)$.

Iterative Breadth-First Search (BFS)

Queue: A FIFO List

- Both add and remove are $O(1)$
 - Add at end of LinkedList
 - Remove from front of LinkedList

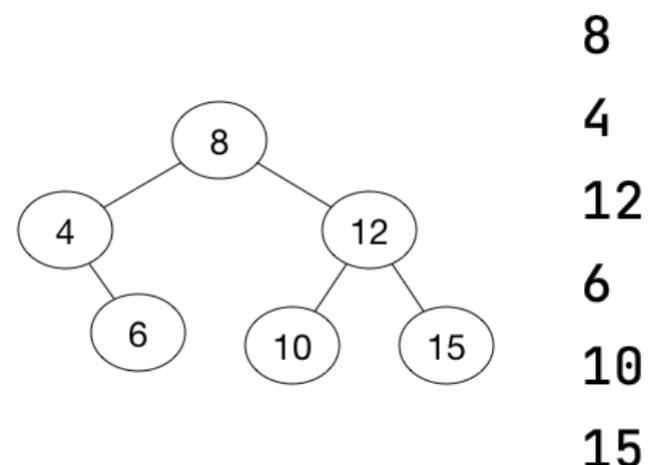
LinkedList implements the Queue interface.

```
5  public static void qdemo() {  
6      String[] strs = {"compsci", "is", "wonderful"};  
7      Queue<String> q = new LinkedList<>();  
8      for(String s : strs) {  
9          q.add(s);  
10     }  
11     while (! q.isEmpty()) {  
12         System.out.println(q.remove());  
13     }  
14 }
```

compsci
is
wonderful

levelOrder Tree Traversal with a queue

```
public static void levelOrder(TreeNode tree) {  
    Queue<TreeNode> queue = new LinkedList<>();  
    queue.add(tree);  
    while (!queue.isEmpty()) {  
        TreeNode current = queue.remove();  
        if (current != null) {  
            System.out.println(current.info);  
            queue.add(current.left);  
            queue.add(current.right);  
        }  
    }  
}
```



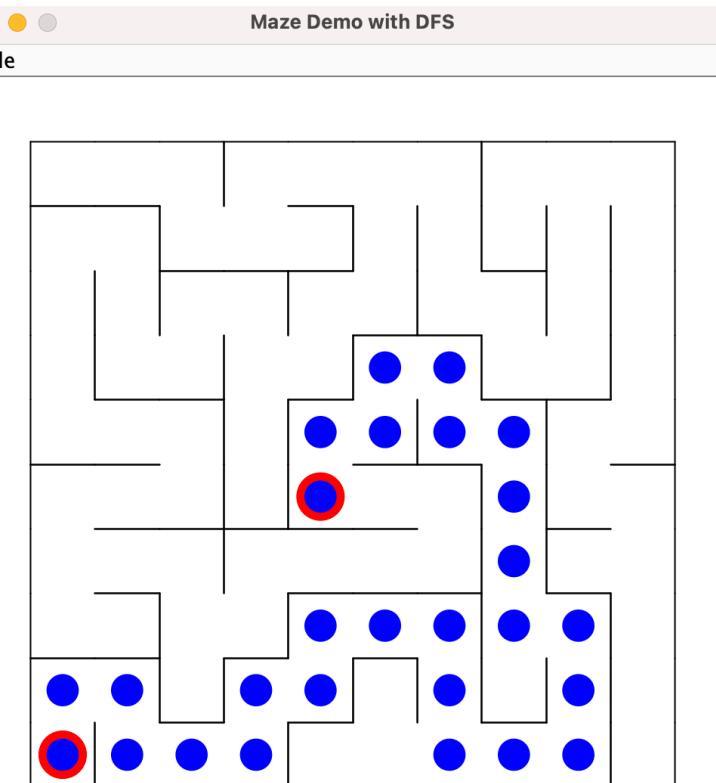
Use a queue to keep track of nodes
First in first out, nodes visited in level order

Depth First Search for Solving Maze

Always explore (recurse on) a new (unvisited) adjacent vertex if possible.

If impossible, **backtrack** to the most recent vertex adjacent to an unvisited vertex and continue.

coursework.cs.duke.edu/cs-201-fall-22/maze-demo

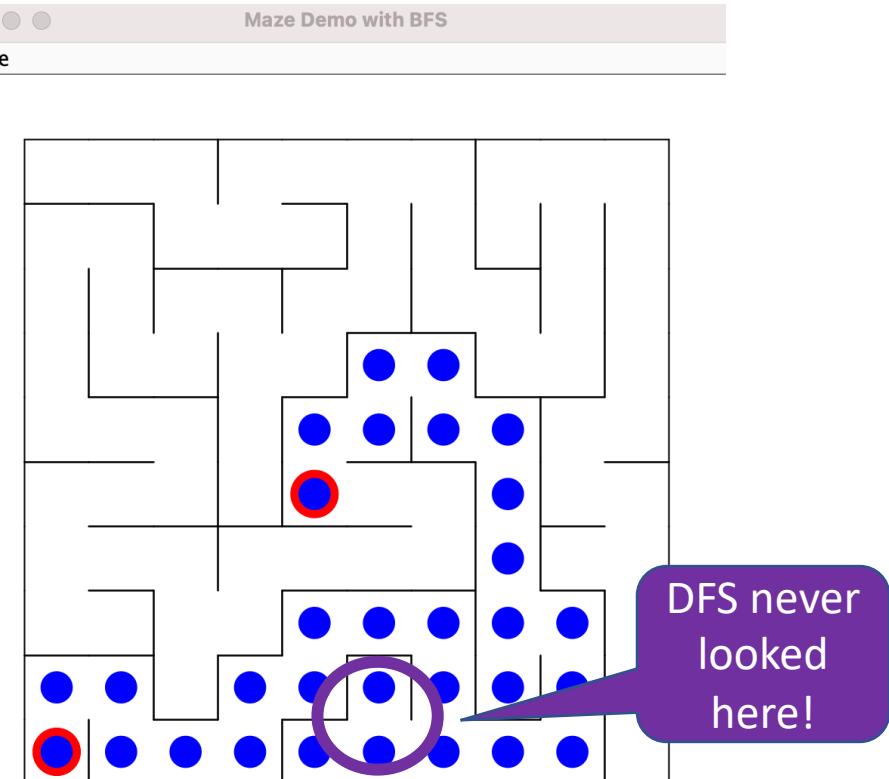


Breadth First Search for Solving Maze

Explore *all* your neighbors (adjacent vertices) before you visit any of your neighbors' neighbors.

Looking for the shortest path/solution.

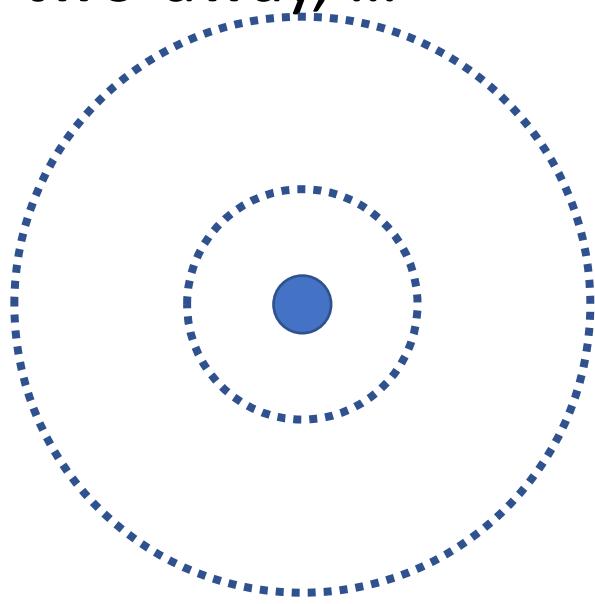
coursework.cs.duke.edu/cs-201-fall-22/maze-demo



Queue = BFS, Stack = DFS

BFS: FIFO Exploration

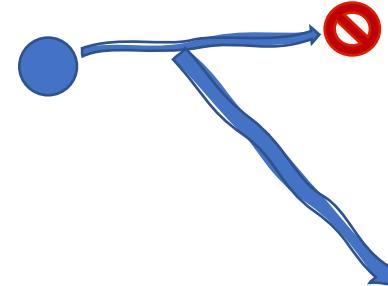
search all locations one-away from start, then two-away, ...



11/16/22

DFS: LIFO Exploration

Search path as far as possible, backtrack if need to another branch...



CompSci 201, Fall 2022, L23: Iterative DFS BFS

30

Initializing Iterative BFS

- **Queue** stores nodes we have *visited/discovered*, but not explored from yet.
- Explore from one *current* node at a time.

```
32  public static void bfs(char start) {  
33      Queue<Character> toExplore = new LinkedList<>();  
34      char current = start;  
35      visited.add(current);  
36      toExplore.add(current);
```

- Queue is FIFI(first-in first-out), so we always explore from the *first/closest (unvisited) node we discovered, breadth-first!*

Iterative BFS Loop

While there are nodes we have not explored from...

Explore from the **closest** discovered node...

```
38  while (!toExplore.isEmpty()) {  
39      current = toExplore.remove();  
40      for (char neighbor : aList.get(current)) {  
41          if (!visited.contains(neighbor)) {  
42              previous.put(neighbor, current);  
43              visited.add(neighbor);  
44              toExplore.add(neighbor);  
45      }  
46  }  
47 }
```

Look at all neighbors of current node...

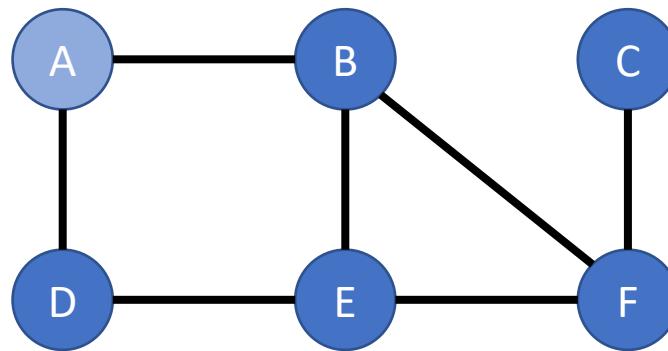
If we haven't seen them before...

Then:

1. note how we got here
2. Note we have seen
3. Mark to explore later

Initialize search at A

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

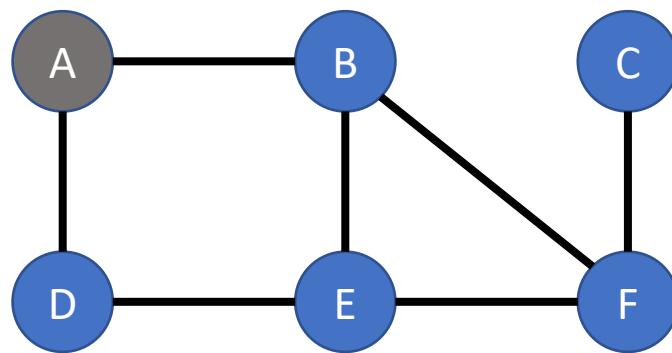
toExplore (queue) **previous (map)** **Visited (set)**

A

{A}

Remove A from the queue

start: A



Adjacency List:

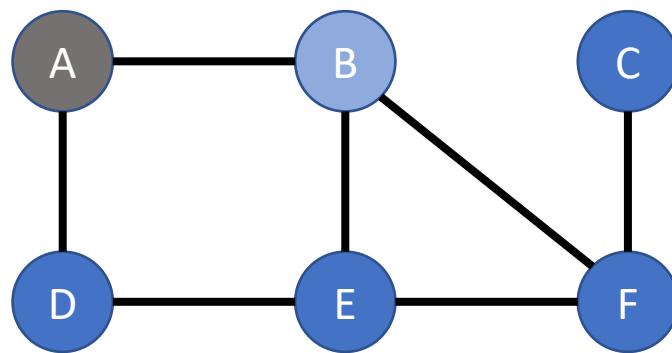
A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (queue) **previous (map)** **Visited (set)**

{A}

Find B from A

start: A



Adjacency List:

A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (queue) **previous (map)** **Visited (set)**

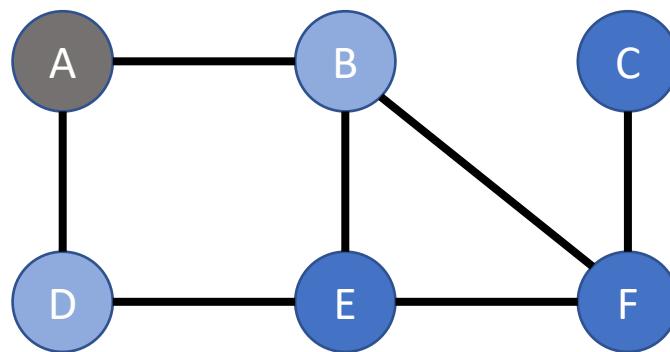
B

B \leftarrow A

{A, B}

Find D from A

start: A



Adjacency List:

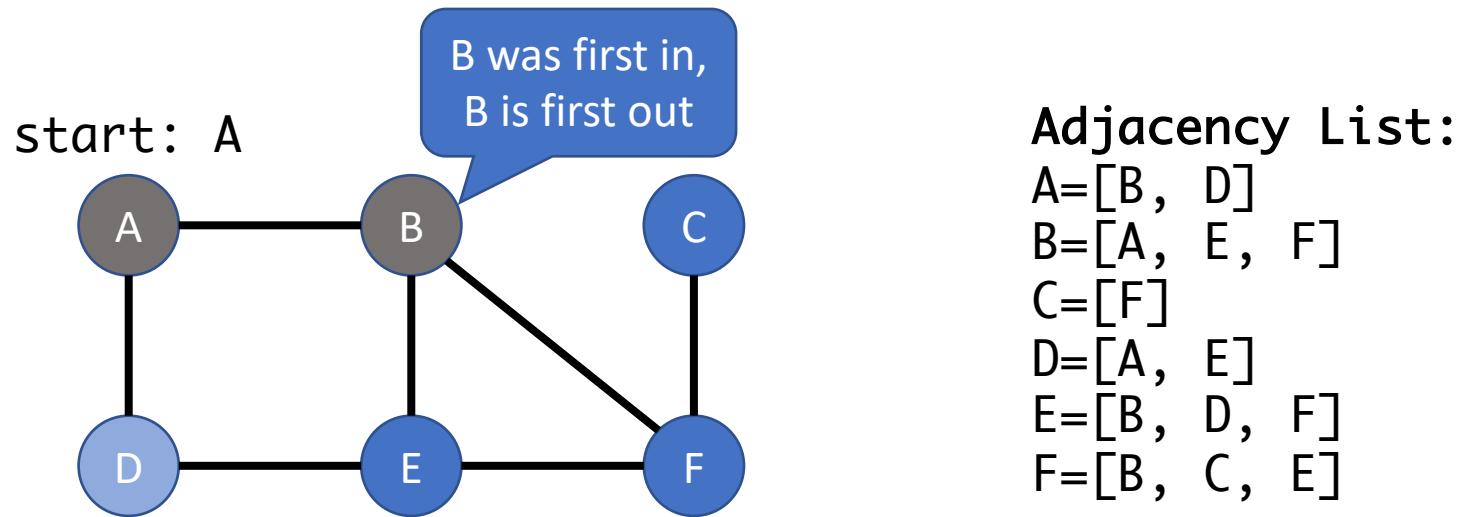
A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (queue) **previous (map)** **Visited (set)**

B	B <- A	{A, B, D}
D	D <- A	

Note the difference,
add to end of queue!

Remove B from queue

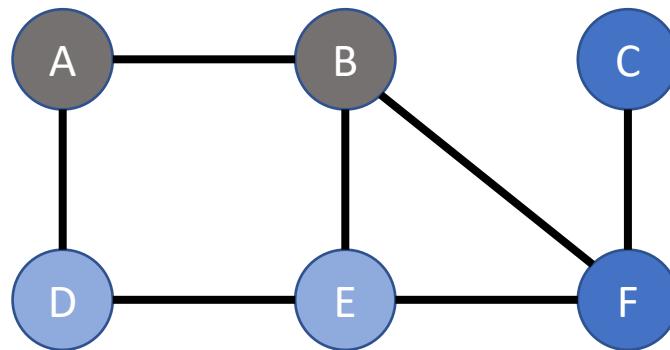


toExplore (queue) **previous (map)** **Visited (set)**

D	B <- A D <- A	{A, B, D}
---	------------------	-----------

Find E from B

start: A



Adjacency List:

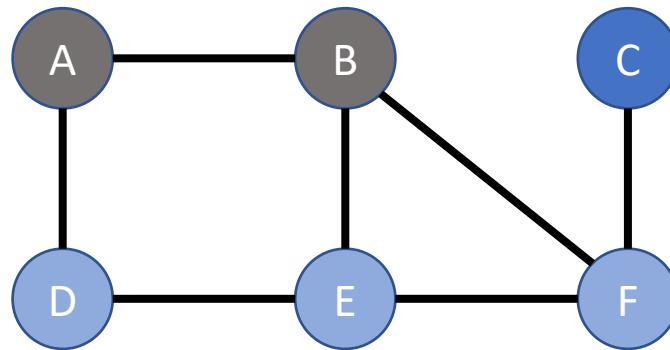
A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (queue) **previous (map)** **Visited (set)**

D	B <- A	{A, B, D, E}
E	D <- A	
	E <- B	

Find F from B

start: A



Adjacency List:

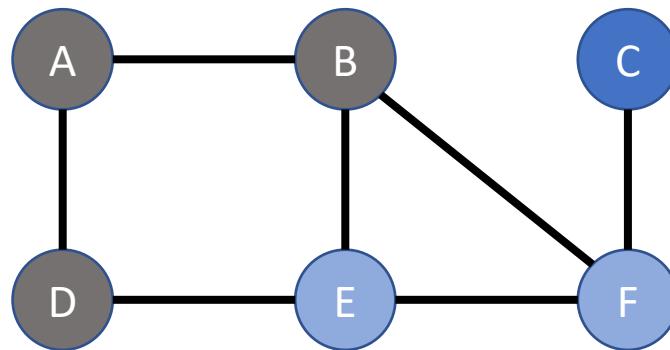
A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (queue) **previous (map)** **Visited (set)**

D	B <- A	{A, B, D, E, F}
E	D <- A	
F	E <- B	
	F <- B	

Remove D from queue

start: A



Adjacency List:

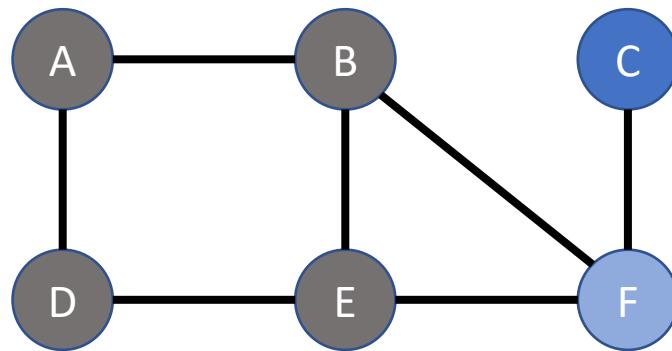
A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (queue) **previous (map)** **Visited (set)**

E	B <- A	{A, B, D, E, F}
F	D <- A	
	E <- B	
	F <- B	

Remove E from queue

start: A



Adjacency List:

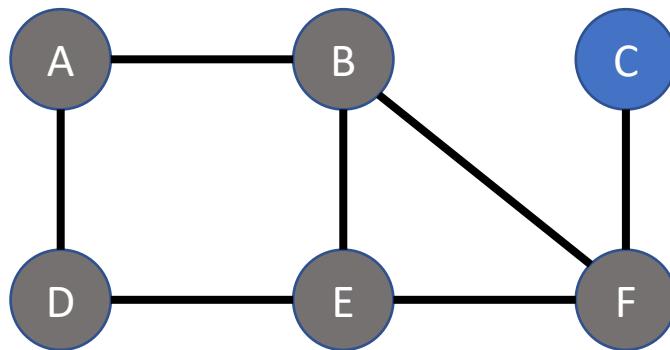
A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (queue) **previous (map)** **Visited (set)**

F	B <- A D <- A E <- B F <- B	{A, B, D, E, F}
---	--------------------------------------	-----------------

Remove F from queue

start: A



Adjacency List:

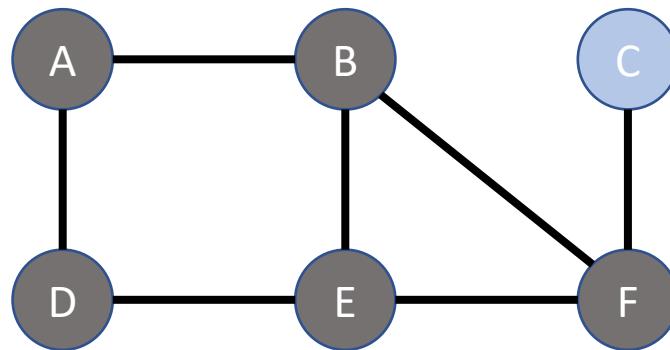
A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (queue) **previous (map)** **Visited (set)**

B <- A	{A, B, D, E, F}
D <- A	
E <- B	
F <- B	

Find C from F

start: A



Adjacency List:

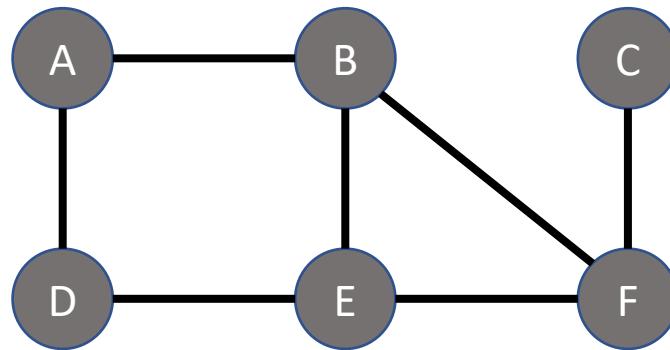
A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (queue) **previous (map)** **Visited (set)**

C	B <- A D <- A E <- B F <- B C <- F	{A, B, D, E, F, C}
---	--	--------------------

Remove C from queue

start: A



Adjacency List:

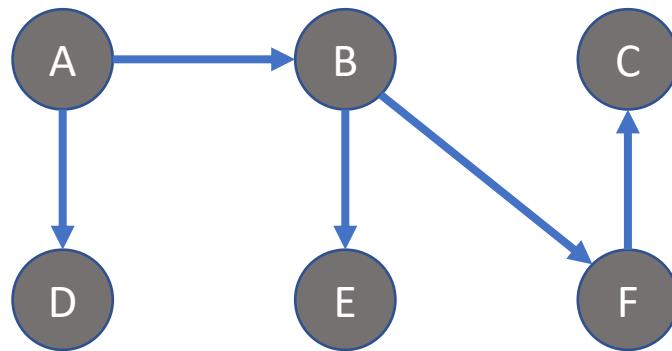
A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (queue) **previous (map)** **Visited (set)**

B <- A	{A, B, D, E, F, C}
D <- A	
E <- B	
F <- B	
C <- F	

BFS Search Tree

start: A



Adjacency List:

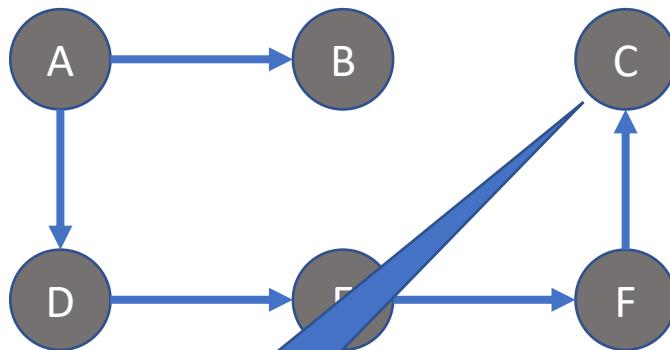
A=[B, D]
B=[A, E, F]
C=[F]
D=[A, E]
E=[B, D, F]
F=[B, C, E]

toExplore (queue) **previous (map)** **Visited (set)**

B <- A	{A, B, D, E, F, C}
D <- A	
E <- B	
F <- B	
C <- F	

Comparing DFS and BFS Search Trees

start: A

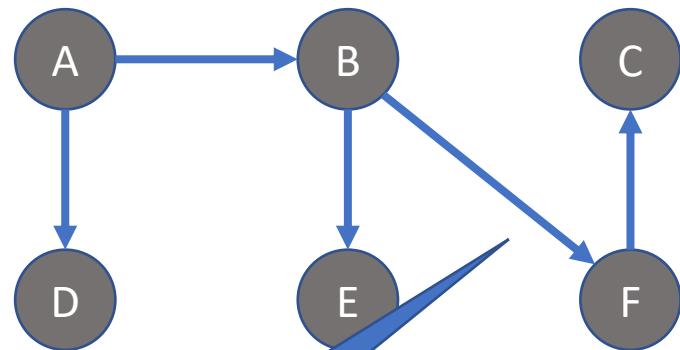


Length 4 path
from A to C

previous (map)

B <- A
D <- A
E <- D
F <- E
C <- F

start: A



Length 3 path
from A to C,
shorter!

previous (map)

B <- A
D <- A
E <- B
F <- B
C <- F

Pathfinding Properties

- DFS and BFS **both** find valid paths to *all* nodes reachable from the start.
 - Can return early if you only want to find a path to a specific target node
- BFS finds the *shortest path* to every reachable node, DFS does *not* guarantee this.

WOTO

Go to duke.is/wjrfp

Not graded for correctness,
just participation.

Try to answer *without* looking
back at slides and notes.

But do talk to your neighbors!

