Index

Introduction to Databases
CompSci 316 Fall 2022
Announcements (Thu. Oct 13)

• Project, project, & project!
  • MS-2 due today (10/13)
  • HW4 due 10/20 - group submission per project team
  • DS7 – team work for project & HW4
Recall the Disk-Main Memory diagram!
Topics

• Index

• Dense vs. Sparse
• Clustered vs. unclustered
• Primary vs. secondary
• Tree-based vs. Hash-index
What are indexes for?

• Given a value, locate the record(s) with this value
  
  SELECT * FROM R WHERE A = value;
  SELECT * FROM R, S WHERE R.A = S.B;

• Find data by other search criteria, e.g.
  
  • Range search
    SELECT * FROM R WHERE A > value;
  
  • Keyword search
Dense and sparse indexes

- **Dense**: one index entry for each search key value
  - One entry may “point” to multiple records (e.g., two users named Jessica)
- **Sparse**: one index entry for each block
  - Records must be clustered according to the search key

When are these possible?

Comparison?
Dense versus sparse indexes

- **Index size**
  - ?

- **Requirement on records**
  - ?

- **Lookup**
  - ?

- **Update**
  - ?
Dense versus sparse indexes

• **Index size**
  • Sparse index is smaller

• **Requirement on records**
  • Records must be clustered for sparse index

• **Lookup**
  • Sparse index is smaller and may fit in memory
  • Dense index can directly tell if a record exists

• **Update**
  • May be easier for sparse index (less movement for updates)
Primary and secondary indexes

• Primary index
  • Created for the primary key of a table
  • Records are usually clustered by the primary key
  • Can be sparse

• Secondary index
  • Usually dense

• SQL
  • PRIMARY KEY declaration automatically creates a primary index, UNIQUE key automatically creates a secondary index
  • Additional secondary index can be created on non-key attribute(s):
    CREATE INDEX UserPopIndex ON User(pop);
What if the index is too big as well?

Sparse index on *uid*

<table>
<thead>
<tr>
<th><em>uid</em></th>
<th>Name</th>
<th>Score</th>
<th>Similarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>123</td>
<td>Milhouse</td>
<td>10</td>
<td>0.2</td>
</tr>
<tr>
<td>142</td>
<td>Bart</td>
<td>10</td>
<td>0.9</td>
</tr>
<tr>
<td>279</td>
<td>Jessica</td>
<td>10</td>
<td>0.9</td>
</tr>
<tr>
<td>345</td>
<td>Martin</td>
<td>8</td>
<td>2.3</td>
</tr>
<tr>
<td>456</td>
<td>Ralph</td>
<td>8</td>
<td>0.3</td>
</tr>
<tr>
<td>512</td>
<td>Nelson</td>
<td>10</td>
<td>0.4</td>
</tr>
<tr>
<td>679</td>
<td>Sherri</td>
<td>10</td>
<td>0.6</td>
</tr>
<tr>
<td>697</td>
<td>Terri</td>
<td>10</td>
<td>0.6</td>
</tr>
<tr>
<td>857</td>
<td>Lisa</td>
<td>8</td>
<td>0.7</td>
</tr>
<tr>
<td>912</td>
<td>Windel</td>
<td>8</td>
<td>0.5</td>
</tr>
<tr>
<td>997</td>
<td>Jessica</td>
<td>8</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Dense index on *name*

- Bart
- Jessica
- Lisa
- Martin
- Milhouse
- Nelson
- Ralph
- Sherri
- Terri
- Windel
What if the index is too big as well?

Sparse index on uid

Dense index on name

Put a another (sparse) index on top of that!
Binary Search Tree

Each node can hold Exactly one entry

Height balanced: All leaves are at the Same level (complete binary tree)

Leaves are sorted

End of Lecture 10/6
(some earlier slides were skipped)
That will be covered next weeks
Remember Terminology

- **Index** search key (key): k
  - Used to search a record

- **Data entry**: k*
  - Pointed to by k
  - Contains record id(s)
    - (-) another level of indirection (+) small and fixed length entries
    - or record itself
      - (-) can be large, cannot be stored in memory, (+) saves some disk access

- **Records or data**
  - Actual tuples
  - Pointed to by record ids

INDEX does this

On disk
B-tree: Generalizing Binary Search Trees

Each node can hold multiple entries, has fixed max size and is sorted.

To be full:
#pointers = #entries + 1

Height balanced

Leaves are sorted
**B⁺-tree: Data only at leaves**

Index Nodes Containing Index entries

Data values can be repeated as index

Leaves are linked

Data entries: Pointers to actual tuples
B⁺-tree balancing properties

• Height constraint: all leaves at the same lowest level
• Fan-out constraint: all nodes at least half full (except root)

<table>
<thead>
<tr>
<th></th>
<th>Max # pointers</th>
<th>Max # keys</th>
<th>Min # active pointers</th>
<th>Min # keys</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-leaf</td>
<td>f</td>
<td>f – 1</td>
<td>[f/2]</td>
<td>[f/2] – 1</td>
</tr>
<tr>
<td>Root</td>
<td>f</td>
<td>f – 1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Leaf</td>
<td>f</td>
<td>f – 1</td>
<td>[f/2]</td>
<td>[f/2]</td>
</tr>
</tbody>
</table>

Check yourself
B⁺-tree: Closer Look

- A hierarchy of nodes with intervals
- Balanced (more or less): good performance guarantee
- Disk-based: one node per block; large fan-out

Max fan-out: 4
Sample B⁺-tree nodes

Non-leaf

Max fan-out: 4

120
150
180

to keys
100 ≤ k

to keys
120 ≤ k < 150

to keys
150 ≤ k < 180

to keys
180 ≤ k

Leaf

120
130

to next leaf node in sequence

to records with these k values;
or, store records directly in leaves (pros/cons?)
Questions

Why do we use B⁺-tree as database index instead of binary trees?

Why do we use B⁺-tree as database index instead of B-trees (next slide)?

- What are the differences/pros/cons of B-trees vs. B⁺-tree as index?
B\(^+\)-tree versus B-tree

• B-tree: why not store records (or record pointers) in non-leaf nodes?
  • These records can be accessed with fewer I/O’s

• Problems?
  • Storing more data in a node decreases fan-out and increases \( h \)
  • Records in leaves require more I/O’s to access
  • Vast majority of the records live in leaves!
Lookups

- SELECT * FROM R WHERE $k = 179$;
- SELECT * FROM R WHERE $k = 32$;

Max fan-out: 4
Practice Problems

- SELECT * FROM R WHERE \( k = 179 \);
- SELECT * FROM R WHERE \( k = 32 \);

Assumptions: Cost = 3
1. Height = 3
2. Each node = 1 block
   = 1 disk I/O = 1 cost
3. All nodes of B+tree on disk
4. At most one matching tuple for every search key
   Just find if such a tuple exist

Assumptions: Cost = 1
1. Height = 3
2. Each node = 1 block
   = 1 disk I/O = 1 cost
3. First two levels are in memory
4. At most one matching tuple for every search key
   Give me the tuple as well

Not found
Recap: Search key and Data entry

- SELECT * FROM R WHERE \( k = 179 \);
Range query

• SELECT * FROM R WHERE \( k > 32 \) AND \( k < 179 \);
Practice Problem 2

- SELECT * FROM R WHERE \( k > 32 \) AND \( k < 179 \)

Cost = \( h + L - 1 \)

Look up 32...

And follow next-leaf pointers until you hit upper bound

Assume height = \( h \)
Assume there are \( L \) matching leaves
= matching data entries reside on \( L \) leaves
Assume one node = one block
= 1 cost = 1 I/O
Everything on disk
Assume you only need to check existence

End of lecture
Thurs 10/13
Insertion

• Insert a record with search key value 32

Look up where the inserted key should go...

And insert it right there
Another insertion example

- Insert a record with search key value 152

```
  100
  /|
  / |
 101 110

 120 150 180
```

Max fan-out: 4

You could reorganize with a sibling here since they have space e.g., (150, 152, 156) and (179, 180, 200)
The parent should be (120, 150, 179)

Oops, node is already full!

What are our options here?
Node splitting

Note:
1. we “copy up” while splitting leaves – Insertion both at leaf and parent
2. The value inserted at parent may *not* be the new value we are inserting

Max fan-out: 4

Oops, that node becomes full!

Need to add to parent node a pointer to the newly created node
More node splitting

• In the worst case, node splitting can “propagate” all the way up to the root of the tree (not illustrated here)
  • Splitting the root introduces a new root of fan-out 2 and causes the tree to grow “up” by one level

Note:
We “push up” while splitting non-leaves, insertion ONLY at the parent node (from the middle)
This is so that we do not have a dangling pointer at non-leaves
Deletion

• Delete a record with search key value 130

Max fan-out: 4

Look up the key to be deleted...

And delete it

Oops, node is too empty!

If a sibling has more than enough keys, steal one!
Stealing from a sibling

Remember to fix the key in the least common ancestor of the affected nodes.
Another deletion example

• Delete a record with search key value 179

Max fan-out: 4

Cannot steal from siblings

Then coalesce (merge) with a sibling!
Coalescing

• Deletion can “propagate” all the way up to the root of the tree (not illustrated here)
  • When the root becomes empty, the tree “shrinks” by one level
Performance analysis

• How many I/O’s are required for each operation?
  • \( h \), the height of the tree (more or less)
  • Plus one or two to manipulate actual records
  • Plus \( O(h) \) for reorganization (rare if \( f \) is large)
  • Minus one if we cache the root in memory

• How big is \( h \)?
  • Roughly \( \log_{\text{fanout}} N \), where \( N \) is the number of records
  • \( B^+ \)-tree properties guarantee that fan-out is least \( f/2 \) for all non-root nodes
  • Fan-out is typically large (in hundreds)—many keys and pointers can fit into one block
  • A 4-level \( B^+ \)-tree is enough for “typical” tables
B⁺-tree in practice

• Complex reorganization for deletion often is not implemented (e.g., Oracle)
  • Leave nodes less than half full and periodically reorganize

• Most commercial DBMS use B⁺-tree instead of hashing-based indexes because B⁺-tree handles range queries
  • A key difference between hash and tree indexes!
Clustered vs. Unclustered Index

• If order of data records in a file is the same as, or `close to’, order of data entries in an index, then clustered, otherwise unclustered

• How does it affect # of page accesses? (in class)
Clustered vs. Unclustered Index

• How does it affect # of page accesses?
  • Recall disk-memory diagram!

• SELECT * FROM USER WHERE age = 50
  • Assume 12 users with age = 50
  • Assume one data page can hold 4 User tuples
  • Suppose searching for a data entry requires 3 IOs in a B+-tree, which contain pointers to the data records (assume all matching pointers = data entries are in the same node of B+-tree)

• What happens if the index is unclustered? (cost 3+12)
• What happens if the index is clustered? (cost <= 3 +(3 +1))
  • +1 for page boundary
The Halloween Problem

• Story from the early days of System R...

    UPDATE Payroll
    SET salary = salary * 1.1
    WHERE salary <= 25000;

    • There is a B⁺-tree index on Payroll(salary)
    • All employees end up earning >= 25000 (why?)

• Solutions?

    • Scan index in reverse, or
    • Before update, scan index to create a “to-do” list, or
    • During update, maintain a “done” list, or
    • Tag every row with transaction/statement id

**ISAM**

- **ISAM (Index Sequential Access Method)**, static version of B+-tree
- Updates are handled by (long) overflow chains
  - Overflow chains and empty data blocks degrade performance
    - Worst case: most records go into one long chain, so lookups require scanning all data!
Beyond ISAM, B-trees, and $B^+$-trees

• Other tree-based indexes: R-trees and variants, GiST, etc.

• Hashing-based indexes: extensible hashing, linear hashing, etc.

• Text indexes: inverted-list index, suffix arrays, etc.

• Other tricks: bitmap index, bit-sliced index, etc.
Hash vs. Tree Index

- Hash indexes can only handle equality queries
  - SELECT * FROM R WHERE age = 5 (requires hash index on (age))
  - SELECT * FROM R, S WHERE R.A = S.A (requires hash index on R.A or S.A)
  - SELECT * FROM R WHERE age = 5 and name = ‘Bart’ (requires hash index on (age, name))

- Index on prefixes: There can be “composite” hash or tree index on a set of attributes.
  - E.g., a tree-index on composite attributes (A, B) may have values as data entries (2, 1), (2, 2) (3, 1), (3, 5), (3, 7), (4, 1), (4, 5), ...
  - Like “lexicographic order” – when same value of A, sort by B

- **(-) Hash index** Cannot handle range queries or prefixes
  - SELECT * FROM R WHERE age >= 5
  - need to use tree indexes (more common)
  - Tree index on (age), or (age, name) works, but not (name, age) – why?
  - Hash index on only (age) works, hash index on (age, name) does not work

- **(+) Hash-indexes are more amenable to parallel processing**
  - Will learn more in hash-based join

- Performance depends on how good the hash function is (whether the hash function distributes data uniformly and whether data has skew)
Trade-offs for Indexes

• Should we use as many indexes as possible?
Trade-offs for Indexes

• Should we use as many indexes as possible?

• Indexes can make
  • queries go faster
  • updates slower

• Require disk space, too