# SQL: Recursion

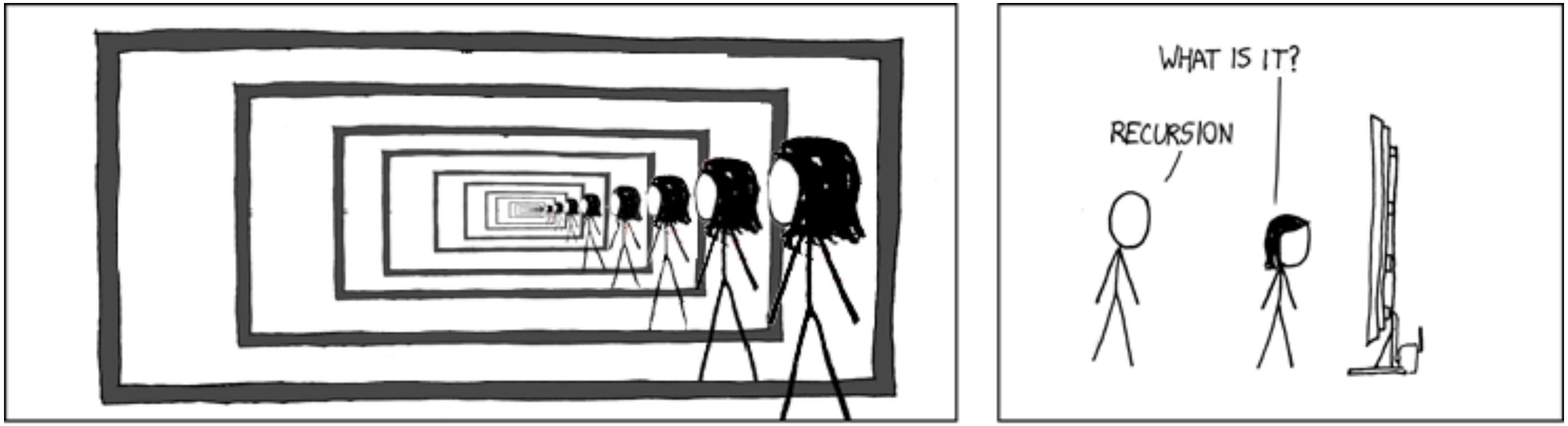
Introduction to Databases

CompSci 316 Fall 2022

DUKE
COMPUTER SCIENCE

# Announcements (Thu., Nov 30)

- Gradiance due Friday 12/2 10 pm
- Work on your projects – check out Ed post
- Check out practice problems and exams on Sakai

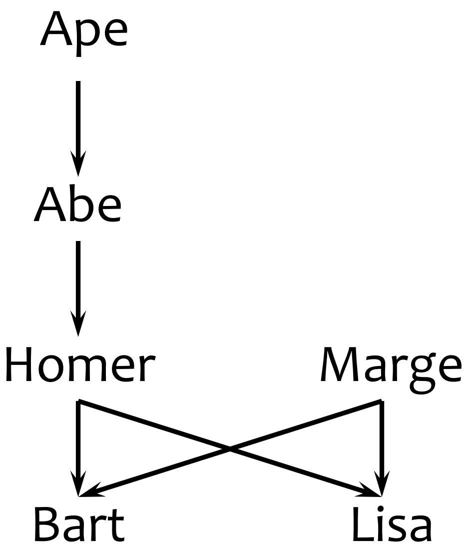


http://xkcdsw.com/1105

# A motivating example

*Parent (parent, child)*

| *parent* | *child* |
|---|---|
| Homer | Bart |
| Homer | Lisa |
| Marge | Bart |
| Marge | Lisa |
| Abe | Homer |
| Ape | Abe |



- Example: find Bart's ancestors
- "Ancestor" has a recursive definition
  - $X$ is $Y$'s ancestor if
    - $X$ is $Y$'s parent, or
    - $X$ is $Z$'s ancestor and $Z$ is $Y$'s ancestor

# Recursion in SQL

- SQL2 had no recursion
  - You can find Bart's parents, grandparents, great grandparents, etc.

```
SELECT p1.parent AS grandparent
FROM Parent p1, Parent p2
WHERE p1.child = p2.parent
AND p2.child = 'Bart';
```

  - But you cannot find all his ancestors with a single query
- SQL3 introduces recursion
  - WITH clause
  - Implemented in PostgreSQL (common table expressions)

# Ancestor query in SQL3

```
WITH RECURSIVE
Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
SELECT anc
FROM Ancestor
WHERE desc = 'Bart';
```

*base case*

*recursion step*

Define
a relation
recursively

Query using the relation
defined in WITH clause

# Finding ancestors

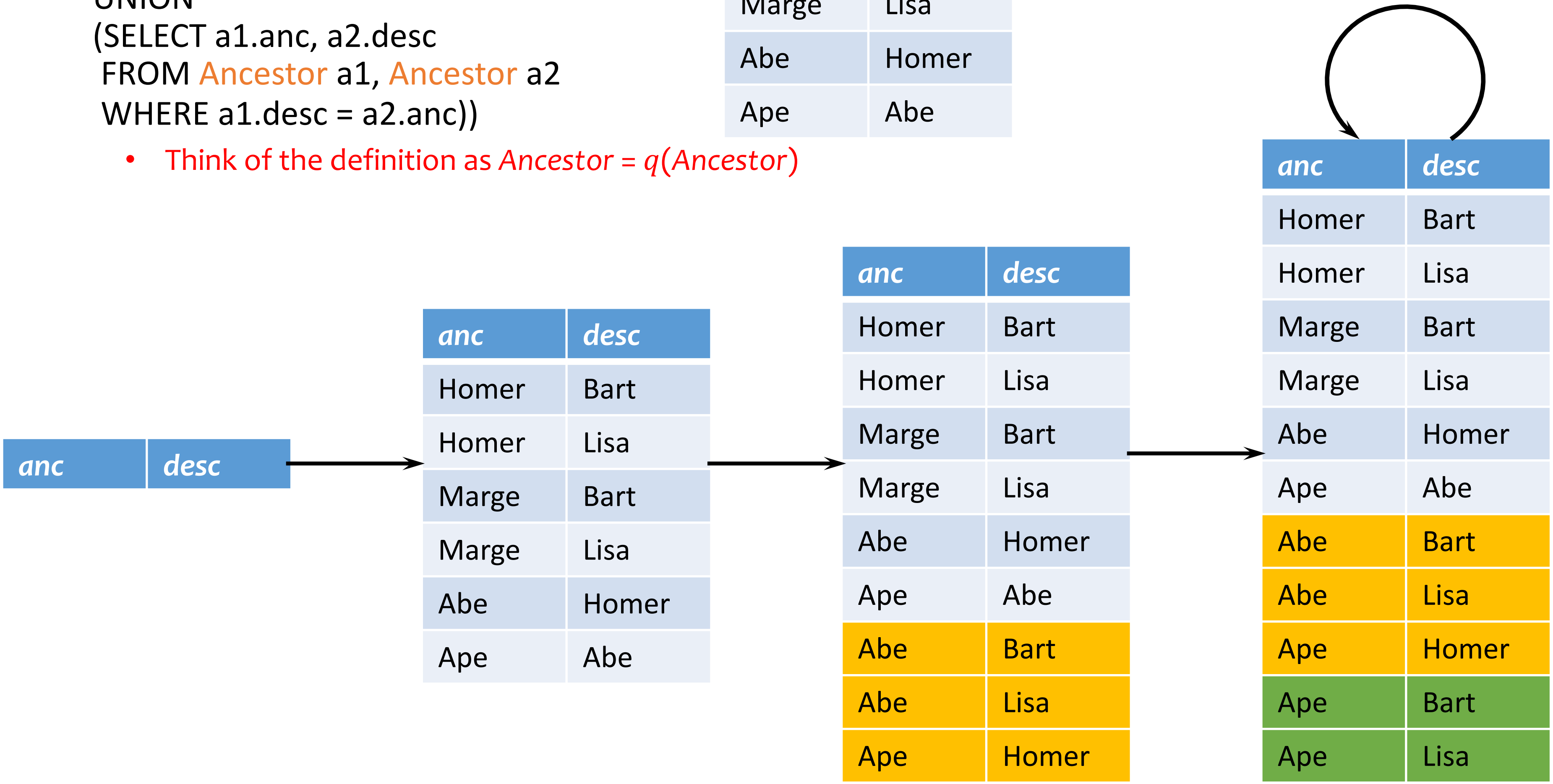
- “Fixed point”
- Start with $\text{Ancestor}_0 = \emptyset$
- Apply the query Q again and again, i.e.,
  **$Q(\text{Ancestor}_{T-1}) = \text{Ancestor}_T$**
- **Until $Q(\text{Ancestor}_T) = \text{Ancestor}_T$**

i.e., no change

- If Q is monotone, unique fixpoint

| *parent* | *child* |
|---|---|
| Homer | Bart |
| Homer | Lisa |
| Marge | Bart |
| Marge | Lisa |
| Abe | Homer |
| Ape | Abe |

- WITH RECURSIVE
  Ancestor(anc, desc) AS
  ((SELECT parent, child FROM Parent)
  UNION
  (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
  - Think of the definition as *Ancestor* = *q*(*Ancestor*)



| *anc* | *desc* |
|---|---|

| *anc* | *desc* |
|---|---|
| Homer | Bart |
| Homer | Lisa |
| Marge | Bart |
| Marge | Lisa |
| Abe | Homer |
| Ape | Abe |

| *anc* | *desc* |
|---|---|
| Homer | Bart |
| Homer | Lisa |
| Marge | Bart |
| Marge | Lisa |
| Abe | Homer |
| Ape | Abe |
| Abe | Bart |
| Abe | Lisa |
| Ape | Homer |

| *anc* | *desc* |
|---|---|
| Homer | Bart |
| Homer | Lisa |
| Marge | Bart |
| Marge | Lisa |
| Abe | Homer |
| Ape | Abe |
| Abe | Bart |
| Abe | Lisa |
| Ape | Homer |
| Ape | Bart |
| Ape | Lisa |

# Intuition behind fixed-point iteration

- Initially, we know nothing about ancestor-descendent relationships
- In the first step, we deduce that parents and children form ancestor-descendent relationships
- In each subsequent steps, we use the facts deduced in previous steps to get more ancestor-descendent relationships
- We stop when no new facts can be proven

# Linear recursion

- With linear recursion, a recursive definition can make only one reference to itself
- Non-linear

  -
    ```
    WITH RECURSIVE Ancestor(anc, desc) AS
    ((SELECT parent, child FROM Parent)
     UNION
     (SELECT a1.anc, a2.desc
      FROM Ancestor a1, Ancestor a2
      WHERE a1.desc = a2.anc))
    ```

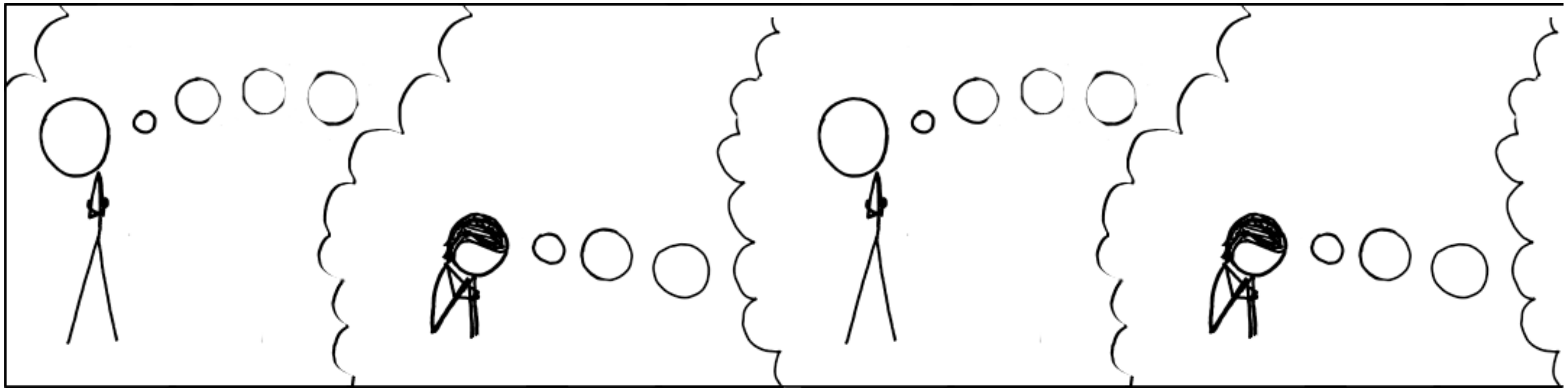
Gives the same answer

- Linear

  -
    ```
    WITH RECURSIVE Ancestor(anc, desc) AS
    ((SELECT parent, child FROM Parent)
     UNION
     (SELECT anc, child
      FROM Ancestor, Parent
      WHERE desc = parent))
    ```

# Linear vs. non-linear recursion

- Linear recursion is easier to implement
  - For linear recursion, just keep joining newly generated *Ancestor* rows with *Parent*
  - For non-linear recursion, need to join newly generated *Ancestor* rows with all existing *Ancestor* rows
- Non-linear recursion may take fewer steps to converge, but perform more work
  - Example: $a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$
  - Linear recursion takes 4 steps
  - Non-linear recursion takes 3 steps
    - More work: e.g., $a \rightarrow d$ has two different derivations



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# Mutual recursion example

- Table *Natural(n)* contains 1, 2, … , 100
- Which numbers are even/odd?
  - An odd number plus 1 is an even number
  - An even number plus 1 is an odd number
  - 1 is an odd number

Order does not matter
Always look at the states of all tables from last step

```
WITH RECURSIVE Even(n) AS
    (SELECT n FROM Natural
     WHERE n = ANY(SELECT n+1 FROM Odd)),
    RECURSIVE Odd(n) AS
    ((SELECT n FROM Natural WHERE n = 1)
     UNION
     (SELECT n FROM Natural
      WHERE n = ANY(SELECT n+1 FROM Even)))
```

Step 0: Odd = {}, Even = {}
Step 1: Odd = {1}, Even = {}Base cas
Step 2: Odd = {1}, Even = {2}
Step 3: Odd = {1, 3}, Even = {2}
Step 4: Odd = {1, 3}, Even = {2, 4}
….
….
Step 100: Odd = {1, 3, … , 99},
Even = {2, 4, …., 100}
Step 101: = Step 100

# Semantics of WITH

- WITH RECURSIVE $R_1$ AS $Q_1$, …,
  RECURSIVE $R_n$ AS $Q_n$
  $Q$;
  - $Q$ and $Q_1, \dots, Q_n$ may refer to $R_1, \dots, R_n$
- Semantics
  1. $R_1 \leftarrow \emptyset, \dots, R_n \leftarrow \emptyset$
  2. Evaluate $Q_1, \dots, Q_n$ using the current contents of $R_1, \dots, R_n$:
     $R_1^{new} \leftarrow Q_1, \dots, R_n^{new} \leftarrow Q_n$
  3. If $R_i^{new} \neq R_i$ for some $i$

     3.1. $R_1 \leftarrow R_1^{new}, \dots, R_n \leftarrow R_n^{new}$

     3.2. Go to 2.
  4. Compute $Q$ using the current contents of $R_1, \dots R_n$
     and output the result

# Mixing negation with recursion

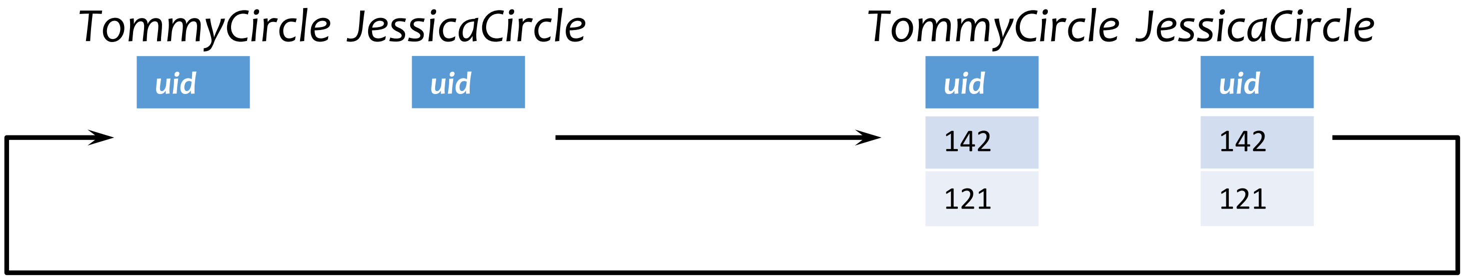
- If $q$ is non-monotone
  - The fixed-point iteration may flip-flop and never converge
  - There could be multiple minimal fixed points—we wouldn't know which one to pick as answer!
- Example: all users join either Jessica's Circle or Tommy's
  - Those not in Jessica's Circle should be in Tom's
  - Those not in Tom's Circle should be in Jessica's
  - 
    ```
    WITH RECURSIVE TommyCircle(uid) AS
        (SELECT uid FROM User WHERE
                  uid NOT IN (SELECT uid FROM JessicaCircle)),
        RECURSIVE JessicaCircle(uid) AS
        (SELECT uid FROM User WHERE
                  uid NOT IN (SELECT uid FROM TommyCircle))
    ```

# Fixed-point iter may not converge

```
WITH RECURSIVE TommyCircle(uid) AS
    (SELECT uid FROM User WHERE
          uid NOT IN (SELECT uid FROM JessicaCircle)),
    RECURSIVE JessicaCircle(uid) AS
    (SELECT uid FROM User WHERE
         uid NOT IN (SELECT uid FROM TommyCircle))
```

Bad query!

| *uid* | *name* | *age* | *pop* |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
| 121 | Allison | 8 | 0.85 |



*TommyCircle*

| *uid* |
|---|

*JessicaCircle*

| *uid* |
|---|

*TommyCircle*

| *uid* |
|---|
| 142 |
| 121 |

*JessicaCircle*

| *uid* |
|---|
| 142 |
| 121 |

# Multiple minimal fixed points

```
WITH RECURSIVE TommyCircle(uid) AS
    (SELECT uid FROM User WHERE
          uid NOT IN (SELECT uid FROM JessicaCircle)),
    RECURSIVE JessicaCircle(uid) AS
    (SELECT uid FROM User WHERE
         uid NOT IN (SELECT uid FROM TommyCircle))
```

Bad query!

| *uid* | *name* | *age* | *pop* |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
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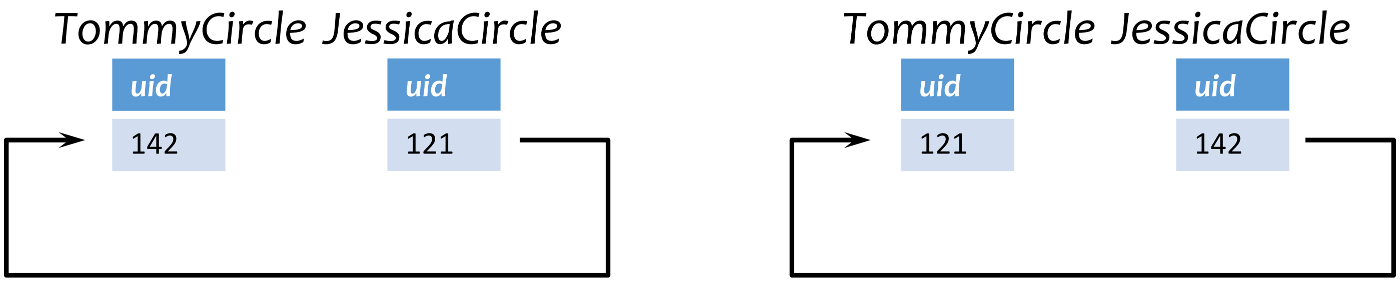


*TommyCircle*

| *uid* |
|---|
| 142 |

*JessicaCircle*

| *uid* |
|---|
| 121 |

*TommyCircle*

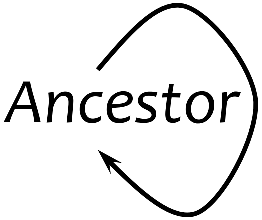
| *uid* |
|---|
| 121 |

*JessicaCircle*

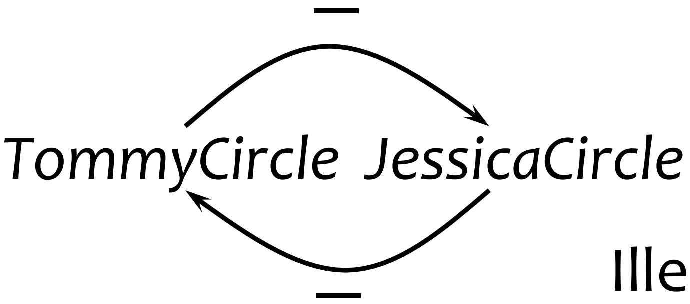
| *uid* |
|---|
| 142 |

# Legal mix of negation and recursion

- Construct a dependency graph
  - One node for each table defined in WITH
  - A directed edge $R \rightarrow S$ if $R$ is defined in terms of $S$
  - Label the directed edge "—" if the query defining $R$ is not monotone with respect to $S$
- Legal SQL3 recursion: no cycle with a "—" edge
  - Called stratified negation
- Bad mix: a cycle with at least one edge labeled "—"



Legal!



Illegal!

# Stratified negation example

- Find pairs of persons with no common ancestors
- Input: Parent(parent, child)

```
WITH RECURSIVE Ancestor(anc, desc) AS
    ((SELECT parent, child FROM Parent) UNION
     (SELECT a1.anc, a2.desc
      FROM Ancestor a1, Ancestor a2
      WHERE a1.desc = a2.anc)),
```

Old ancestor query

```
    Person(person) AS
    ((SELECT parent FROM Parent) UNION
     (SELECT child FROM Parent)),
```

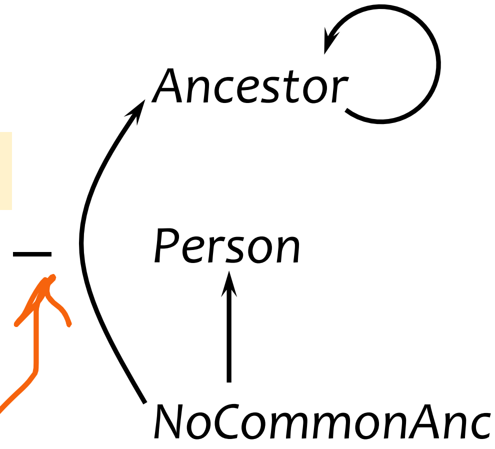
All people in the db

```
    NoCommonAnc(person1, person2) AS
    ((SELECT p1.person, p2.person
      FROM Person p1, Person p2
      WHERE p1.person <> p2.person)
     EXCEPT
     (SELECT a1.desc, a2.desc
      FROM Ancestor a1, Ancestor a2
      WHERE a1.anc = a2.anc))

SELECT * FROM NoCommonAnc;
```

All Pairs of people

Except the people with a common ancestor



*Ancestor*

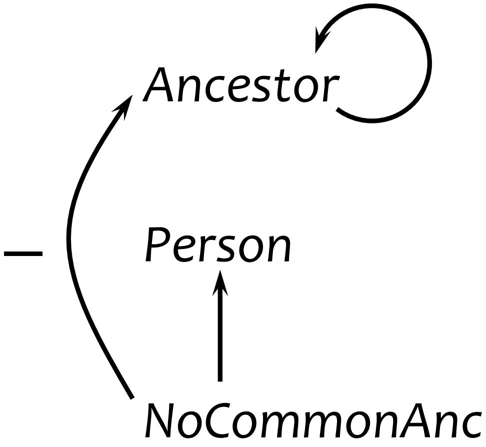
–

*Person*

*NoCommonAnc*

# Evaluating stratified negation

- The stratum of a node $R$ is the maximum number of "—" edges on any path from $R$ in the dependency graph
  - *Ancestor*: stratum 0
  - *Person*: stratum 0
  - *NoCommonAnc*: stratum 1



- Evaluation strategy
  - Compute tables lowest-stratum first
  - For each stratum, use fixed-point iteration on all nodes in that stratum
    - Stratum 0: *Ancestor* and *Person*
    - Stratum 1: *NoCommonAnc*

☞Intuitively, there is no negation within each stratum

# Practice problem: Recursion

- What does this query compute?
- Input: Edge(start, end) denoting directed edges in a graph from u to v. Assume nodes take integer values.

- 
```
WITH RECURSIVE
Mystery(x, y) AS
((SELECT start, end FROM Edge)
 UNION
 (SELECT a1.x, a3.end
  FROM Mystery a1, Edge a2, Edge a3
  WHERE a1.y = a2.start and a2.end=a3.start))
```

```
SELECT y FROM Mystery m1, Mystery m2
WHERE m1.y = m2.x AND m1.x = 5 AND m2.y = 5
```

# Practice problem: Recursion w/ Negation

- Input: Edge(start, end) denoting directed edges in a graph from u to v. Assume nodes take integer values.

Write a query to compute pairs of nodes (x, y) such that there are no paths from x to y

# Practice problem: Recursion w/ Negation

- Input: Edge(start, end) denoting directed edges in a graph from u to v. Assume nodes take integer values.

Write a query to compute pairs of nodes (x, y) such that there are no paths from x to y

# Summary

- SQL3 WITH recursive queries
- Solution to a recursive query (with no negation): unique minimal fixed point
- Computing unique minimal fixed point: fixed-point iteration starting from ∅
- Mixing negation and recursion is tricky
    - Illegal mix: fixed-point iteration may not converge; there may be multiple minimal fixed points
    - Legal mix: stratified negation (compute by fixed-point iteration stratum by stratum)