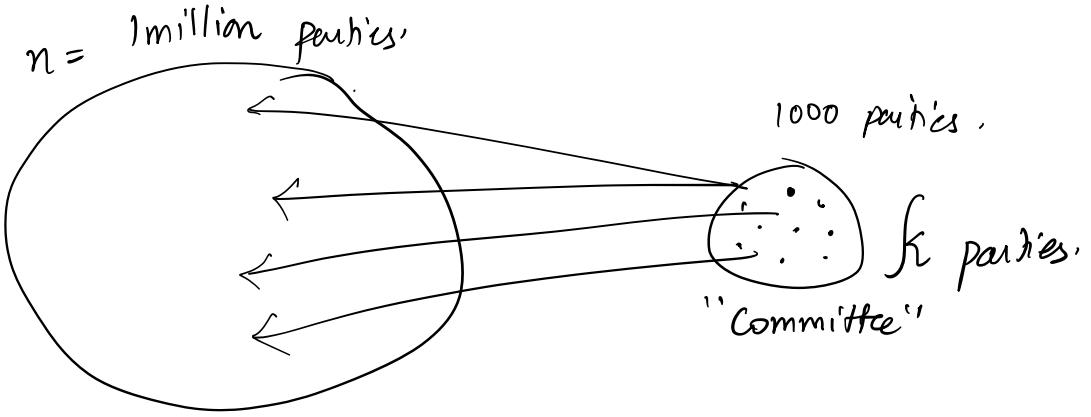
Dolev-Reischuk: Any deterministic protocol requires $O(n^2)$ communication

Idea: One party p that does not receive any message. p will disagree with other honest parties.

Q: Can we obtain a sub-quadratic communication protocol if we assume randomization?



"Every party to every other party": $O(n^2)$ → 10^{12}

"Every party in the committee (k) sends to every other party (n)": $O(kn)$

Coin c. (256 bit) "Common coin".

$H(id_p, c) \Longrightarrow$ if this value is "small enough," then the party is in the committee.

If we set k as the expected committee size, since every party is elected uniformly & independently

at random, we can argue that the committee size $(1-\delta)k <$ committee size $< (1+\delta)k$. except with probability $e^{-\Omega(\delta^2 k)}$. (Chernoff bound).

If we start off with $\frac{1}{3} - \epsilon$ ($\epsilon = 0.05$) Byzantine parties in the entire population, then the committee will have $< \frac{1}{3}$ Byzantine parties.

$2^{-40} \Rightarrow 50$

k.

Protocol! Secure against a static adversary.

- Elect committee C of size k. (no communication)
- Run the agreement protocol within the committee. ($O(k^2)$ communication)
- All parties learn the output by communicating with sufficiently many committee members. ($O(nk)$ communication)

$$O(nk + k^2) = O(nk) =$$

Identities of committee members are public:

- An adversary can attack all committee members
- Bribery.

Adaptivity of the adversary.

Static: Adversary corrupts upto t parties at the beginning of the protocol.

Adaptive: Adversary corrupts parties during the execution.
- A party can be corrupted at the start of a round.

Mobile:

Bitcoin: Static ✓ or adaptively ✓ secure?

"a miner useful now is not useful at a later point":

(A) ↳ I do not know whether I am "in the committee" until I mine PoW.

(B) ↳ (Verify) Once I do win, I can prove to the world, that I am the winner.

(C) ↳ An adversary cannot modify the contents of the block (even by corrupting me).

(D) ↳ (Rand) Randomized / fair.

(B) - (C) - (D).

Verifiable Random function:

$VRF(sk_i, r)$ ⟶ o/p:

r ↓ Prev_hash / Genesis's + round

$VRF_{prove}(sk_i, \ ) \to$ o/p

$VRF_{verify}(\ , pk_i) \Rightarrow$ 1, 0

↳ Uniformly random.

↳ Verifiable.

↳ Unique

→ Change secret keys frequently: Key evolving signatures.

(pk_i, sk_i^x) — Round x.

$\downarrow$ $\downarrow$ $x+1$

(pk_i, sk_i^{x+1})

↳ If I am elected, create a local msg/proposal.

↳ Evolve my key $sk_i^x \rightarrow sk_i^{x+1}$

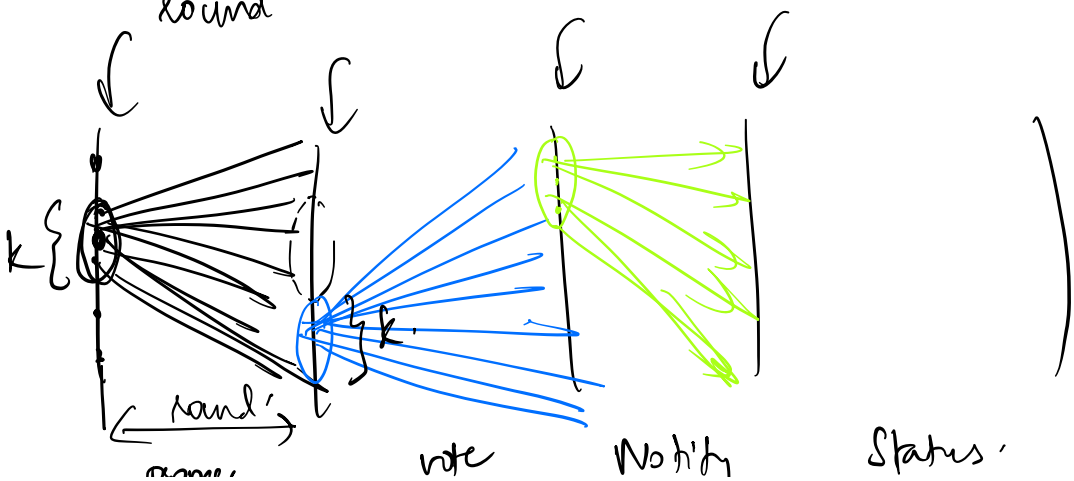
↳ Delete sk_i^x.

↳ Send the proposal to everyone.

Player-replaceability:

↳ Use subquadratic - comm^n protocol for eligibility.

↳ Elect a different committee in each round.



$O(n\,poly(k))$.

Player-replaceability: Sub-quadratic $comm^n$ + adaptive adversary.