Section: Parsing Ch. 15

Parsing: Deciding if x$\in \Sigma^*$ is in L(G) for some CFG G.

Consider the CFG G:

$$\begin{aligned} S &\rightarrow Aa \\ A &\rightarrow AA \mid ABa \mid \lambda \\ B &\rightarrow BBa \mid b \mid \lambda \end{aligned}$$

not good format for parsing

Is ba in L(G)? Running time?

No

takes a long time

New grammar G' is:

better format

$$\begin{aligned} S &\rightarrow Aa \mid a \\ A &\rightarrow AA \mid ABa \mid Aa \mid Ba \mid a \\ B &\rightarrow BBa \mid Ba \mid a \mid b \end{aligned}$$

Is ba in L(G)? Running time?

NO

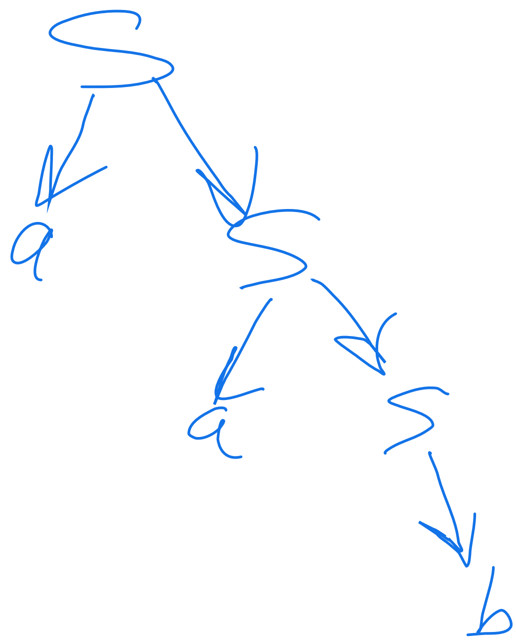
no to stop after twice the length of the string steps

Top-down Parser:

- Start with S and try to derive the string.

$$\mathbf{S \rightarrow aS \mid b}$$

aab

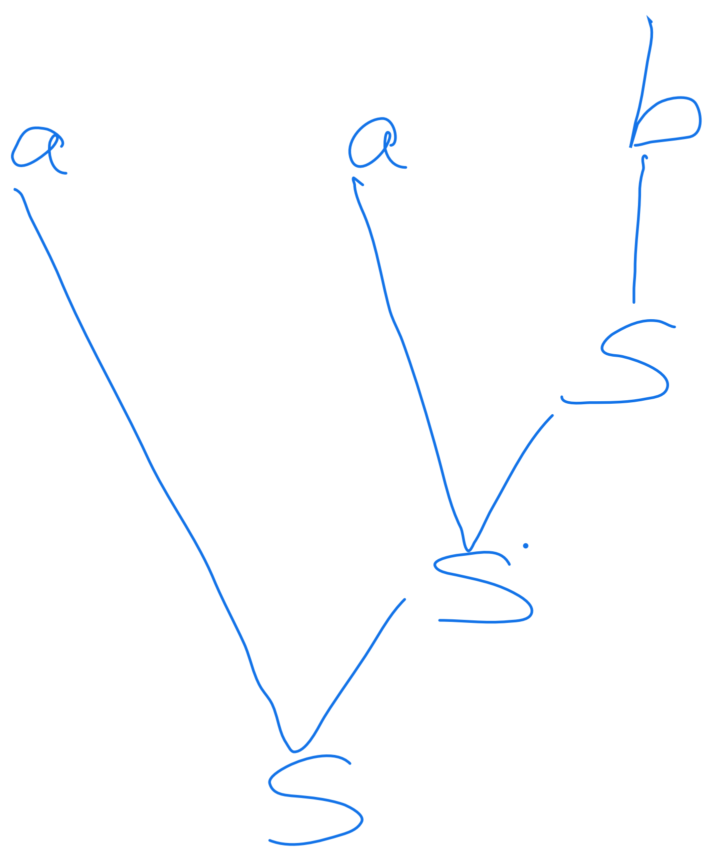


- Examples: LL Parser, Recursive Descent

Bottom-up Parser:

- **Start with string, work backwards, and try to derive S.**

aab



- **Examples: Shift-reduce, Operator-Precedence, LR Parser**

The function FIRST:

$$\mathbf{G=(V,T,S,P)}$$
$$\mathbf{w,v \in (V \cup T)^*}$$
$$\mathbf{a \in T}$$
$$\mathbf{X,A,B \in V}$$
$$\mathbf{X}_I \in \mathbf{(V \cup T)^+}$$

Definition: FIRST

Given a context-free grammar G = (V, T, S, P), $a \in T$ and $w, v \in (V \cup T)^*$, the FIRST(w) is the set of terminals that can be the first terminal a in $w \overset{*}{\Rightarrow} av$. λ is in FIRST(w) if $w \overset{*}{\Rightarrow} \lambda$.

We show how to calculate FIRST for variables and terminals in the grammar, for λ and for strings.

Algorithm for FIRST

Given a grammar G=(V, T, S, P), calculate FIRST(w) for w in $(V \cup T)^*$,

1. **For** $a \in T$**,** $FIRST(a) = \{a\}$**.**
2. **FIRST(λ) = $\{\lambda\}$.**
3. **For** $A \in V$**, set FIRST(A) = $\{\}$.**
4. **Repeat these steps until no more terminals or λ can be added to any FIRST set for variables.**

   For every production $A \rightarrow w$

   FIRST(A) = FIRST(A) $\cup$ FIRST(w)

5. For $w = x_1 x_2 x_3 \ldots x_n$ **where** $x_i \in (V \cup T)$

a) FIRST(w) **= FIRST**(x_1)

b)

For i **from 2 to** n **do:**

    if $x_j \stackrel{*}{\Rightarrow} \lambda$ **for all** j **from** 1 **to** $i - 1$ **then**

        FIRST(w) **= FIRST**$(w) \cup$ **FIRST**(x_i) **-** $\{\lambda\}$

c)

If $x_i \stackrel{*}{\Rightarrow} \lambda$ **for all** i **from** 1 **to** n **then**

    FIRST(w) **= FIRST**$(w) \cup \{\lambda\}$

Example:

$$\mathbf{S} \rightarrow \mathbf{aSc} \mid \mathbf{B}$$
$$\mathbf{B} \rightarrow \mathbf{b} \mid \lambda$$

$\mathbf{FIRST(B)} = \{b, \lambda\}$

$\mathbf{FIRST(S)} = \{b, a, \lambda\}$

$\mathbf{FIRST(Sc)} = \{c, b, a\}$

$Sc \stackrel{*}{\Rightarrow} \lambda$ $Sc \not\Rightarrow \lambda$

$Sc \Rightarrow Bc \Rightarrow c$

Example

$$
\begin{aligned}
\mathbf{S} &\rightarrow \mathbf{BCD} \mid \mathbf{aD} \\
\mathbf{A} &\rightarrow \mathbf{CEB} \mid \mathbf{aA} \\
\mathbf{B} &\rightarrow \mathbf{b} \mid \lambda \\
\mathbf{C} &\rightarrow \mathbf{dB} \mid \lambda \\
\mathbf{D} &\rightarrow \mathbf{cA} \mid \lambda \\
\mathbf{E} &\rightarrow \mathbf{e} \mid \mathbf{fE}
\end{aligned}
$$

$\mathbf{FIRST(S)} = \{a, b, c, d, \lambda\}$

$\mathbf{FIRST(A)} = \{a, d, e, f\}$

$\mathbf{FIRST(B)} = \{b, \lambda\}$

$\mathbf{FIRST(C)} = \{d, \lambda\}$

$\mathbf{FIRST(D)} = \{c, \lambda\}$

$\mathbf{FIRST(E)} = \{e, f\}$

$A \Rightarrow CEB \Rightarrow EB \Rightarrow fE$

Definition: FOLLOW

Given a context-free grammar G = (V, T, S, P)**,** $A \in V$**,** $a \in T$ **and** $w, v \in (V \cup T)^*$**, FOLLOW(**A**) is the set of terminals that can be the first terminal** a **immediately following** A **in some sentential form** $vAaw$**.** $\$$ **is always in FOLLOW(S).**

$\$ \to w\$$

$S\$ \Rightarrow \ldots A \ldots \$ \ldots \$ \Rightarrow w\$$

Algorithm for FOLLOW

To calculate FOLLOW for the variables in G=(V, T, S, P). Let $A, B \in V$ **and** $v, w \in (V \cup T)^*$**.**

1. **\$ is in** $FOLLOW(S)$**.**
2. **For** $A \rightarrow vB$**,** $FOLLOW(A)$ **is in** $FOLLOW(B)$**.**

   $A_- \rightarrow vB_=$

3. **For** $A \rightarrow vBw$**:**

   (a) $FIRST(w) - \{\lambda\}$ **is in** $FOLLOW(B)$**.**

   (b) **If** $\lambda \in FIRST(w)$**, then** $FOLLOW(A)$ **is in** $FOLLOW(B)$**.**

Example:

$$\mathbf{S} \to \mathbf{aSc} \mid \mathbf{B}$$
$$\mathbf{B} \to \mathbf{b} \mid \lambda$$

FOLLOW(S) = {$, c}

FOLLOW(B) = {$, c}

Example:

$$
\begin{aligned}
\mathbf{S} &\rightarrow \mathbf{BCD} \mid \mathbf{aD} \\
\mathbf{A} &\rightarrow \mathbf{CEB} \mid \mathbf{aA} \\
\mathbf{B} &\rightarrow \mathbf{b} \mid \lambda \\
\mathbf{C} &\rightarrow \mathbf{dB} \mid \lambda \\
\mathbf{D} &\rightarrow \mathbf{cA} \mid \lambda \\
\mathbf{E} &\rightarrow \mathbf{e} \mid \mathbf{fE}
\end{aligned}
$$

FOLLOW(S) = $\{\$\}$

FOLLOW(A) = $\{\$\}$

FOLLOW(B) = $\{\$, d, c, e, f\}$

FOLLOW(C) = $\{c, e, f, \$\}$

FOLLOW(D) = $\{\$\}$

FOLLOW(E) = $\{\$, b\}$