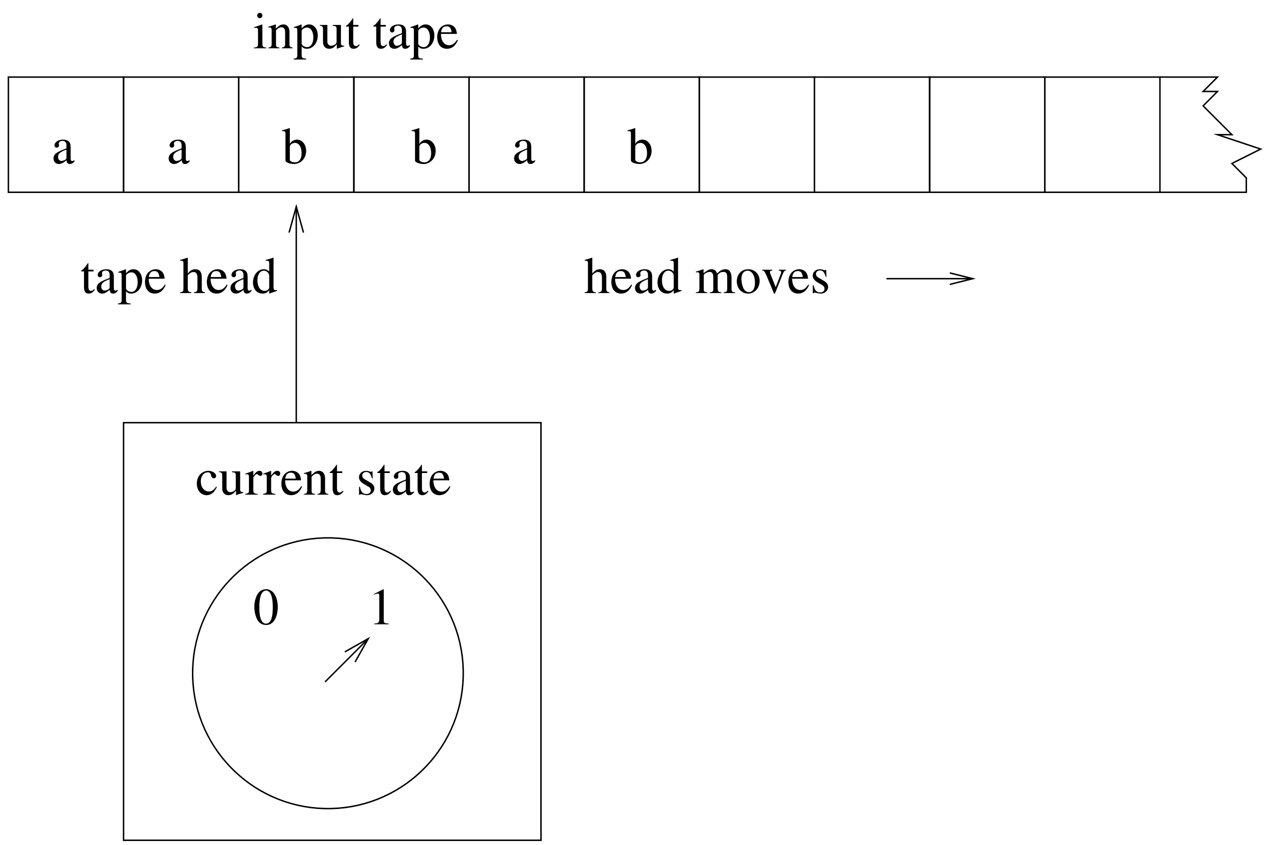
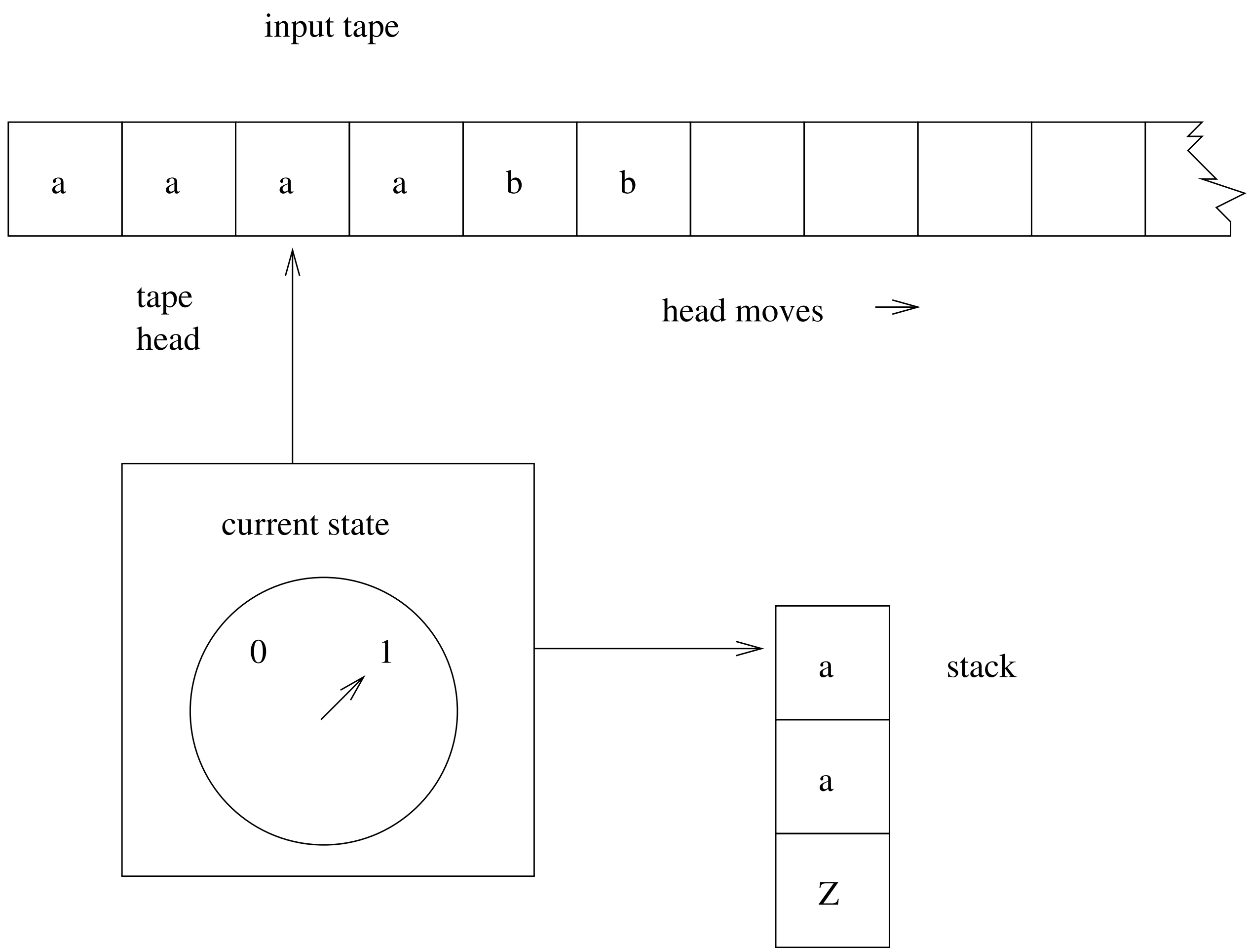
Section: Pushdown Automata

Ch. 7 - Pushdown Automata

A DFA=(Q,Σ,δ,q_0,F)



Modify DFA by adding a stack. New machine is called Pushdown Automata (PDA).



Definition: Nondeterministic PDA (NPDA) is defined by

$$\mathbf{M}=(\mathbf{Q},\Sigma, \Gamma, \delta, q_0, \mathbf{z}, \mathbf{F})$$

where

Q is finite set of states

Σ **is tape (input) alphabet**

Γ **is stack alphabet**

q_0 **is initial state**

z - start stack symbol, $\mathbf{z} \in \Gamma$

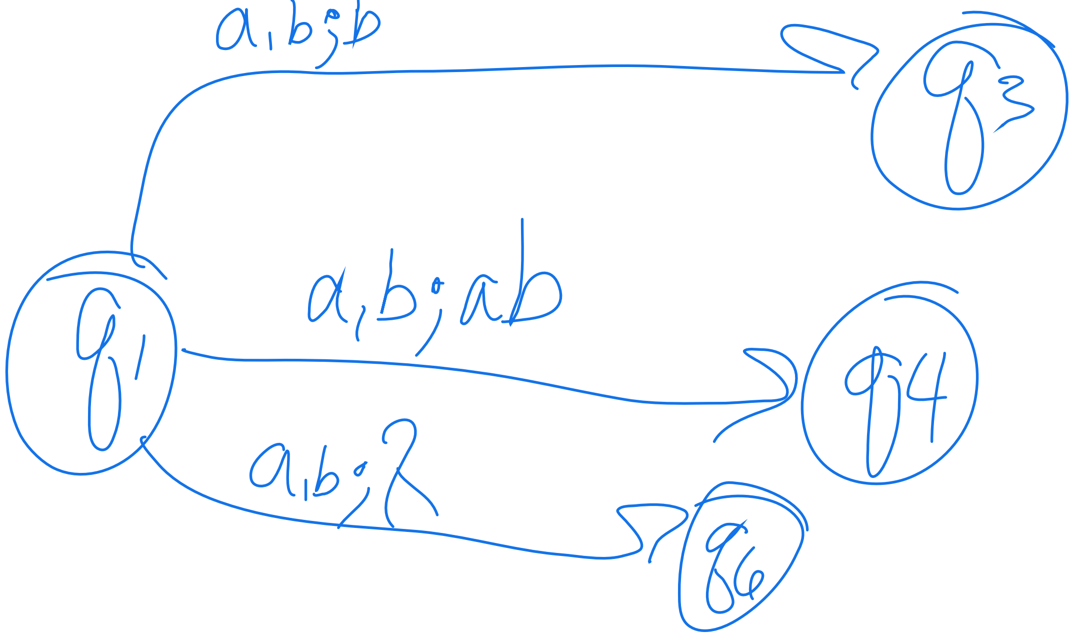
$\mathbf{F} \subseteq \mathbf{Q}$ **is set of final states.**

δ**:**$\mathbf{Q}\times(\Sigma \cup \{\lambda\})\times\Gamma\rightarrow$ **finite subsets of** $Q \times \Gamma^*$

Example of transitions

$$\delta(q_1,\mathbf{a},\mathbf{b}) = \{(q_3,\mathbf{b}),(q_4,\mathbf{ab}),(q_6,\lambda)\}$$

The diagram for the above transitions is:



Instantaneous Description:

$$(\mathbf{q},\mathbf{w},\mathbf{u})$$

freeze current state of the machine

on the stack

still to read

Description of a Move:

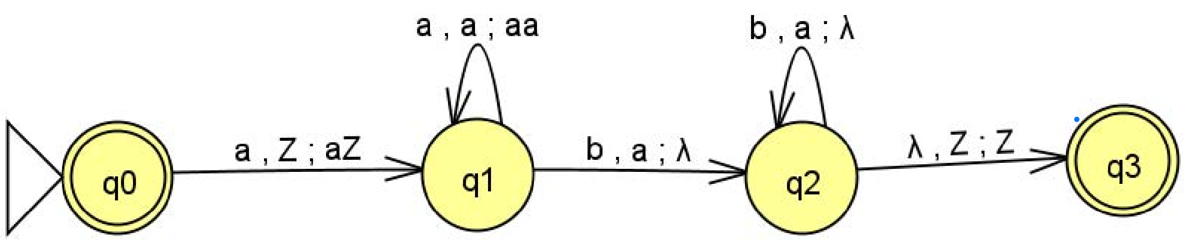
$$(q_1,\mathbf{aw},\mathbf{bx}) \vdash (q_2,\mathbf{w},\mathbf{yx})$$

iff

$$(q_2, y) \in \delta(q_1, a, b)$$

Definition Let M=(Q,Σ,Γ,δ,q_0,z,F) be a NPDA. L(M)=$\{$w$\in \Sigma^*$ | (q_0,w,z) $\overset{*}{\vdash}$ (p,λ,u), p$\in$F, u$\in \Gamma^*\}$. The NPDA accepts all strings that start in q_0 and end in a final state.

Example: L=$\{a^n b^n | n \geq 0\}$**,** $\Sigma = \{a, b\}$**,** $\Gamma = \{z, a\}$

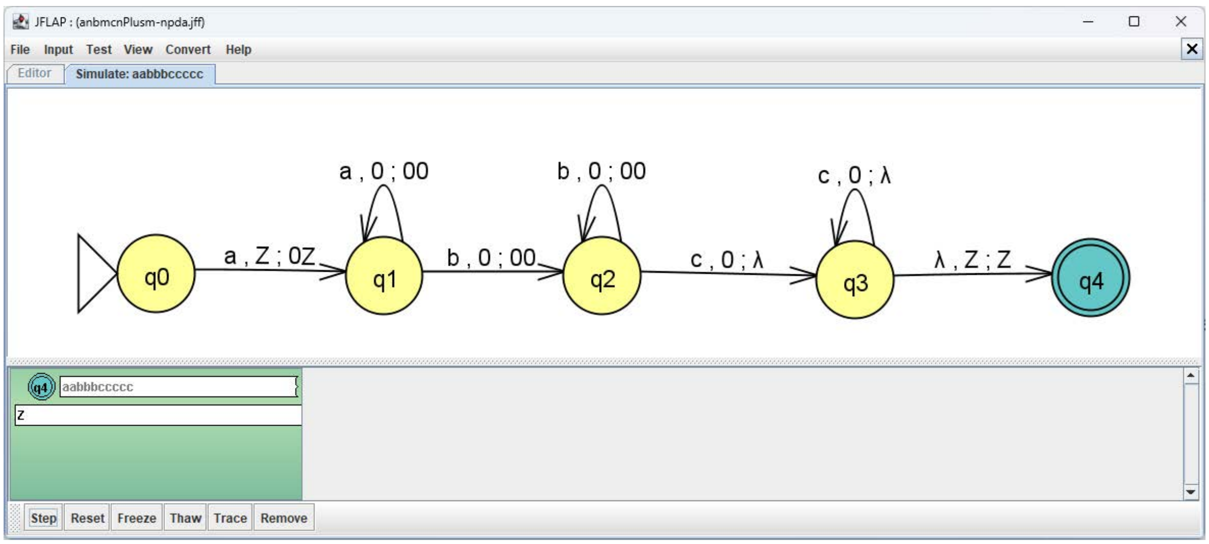


Another Definition for Language Acceptance

NPDA M accepts L(M) by empty stack:

$$\mathbf{L(M)}=\{w \in \Sigma^* | (q_0, w, z) \overset{*}{\vdash} (p, \lambda, \lambda)\}$$

Example: $\mathbf{L}=\{a^n b^m c^{n+m} | n, m > 0\}$,
$\Sigma = \{a, b, c\}$, $\Gamma = \{0, z\}$



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Examples for you to try on your own: (solutions are at the end of the handout).

- **L=**$\{a^n b^m | m > n, m, n > 0\}$**,** $\Sigma = \{a, b\}$**,** $\Gamma = \{z, a\}$
- **L=**$\{a^n b^{n+m} c^m | n, m > 0\}$**,** $\Sigma = \{a, b, c\}$**,**
- **L=**$\{a^n b^{2n} | n > 0\}$**,** $\Sigma = \{a, b\}$

Definition: A PDA M=(Q,Σ,Γ,δ,q_0,z,F) is *deterministic* **if for every** $q \in$**Q,** $a \in \Sigma \cup \{\lambda\}$**,** $b \in \Gamma$

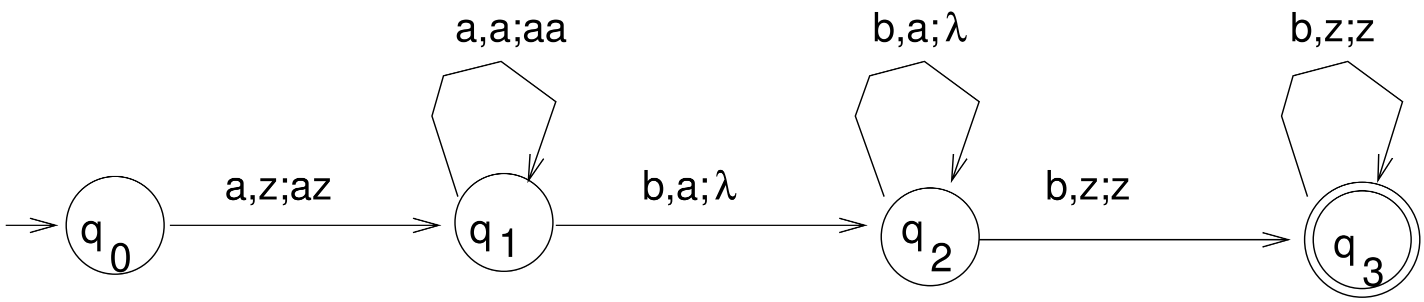
1. $\delta(q, a, b)$ **contains at most 1 element**
2. **if** $\delta(q, \lambda, b) \neq \emptyset$ **then** $\delta(q, c, b)$=$\emptyset$ **for all** $c \in \Sigma$

Definition: L is DCFL iff $\exists$ **DPDA M s.t. L=L(M).**

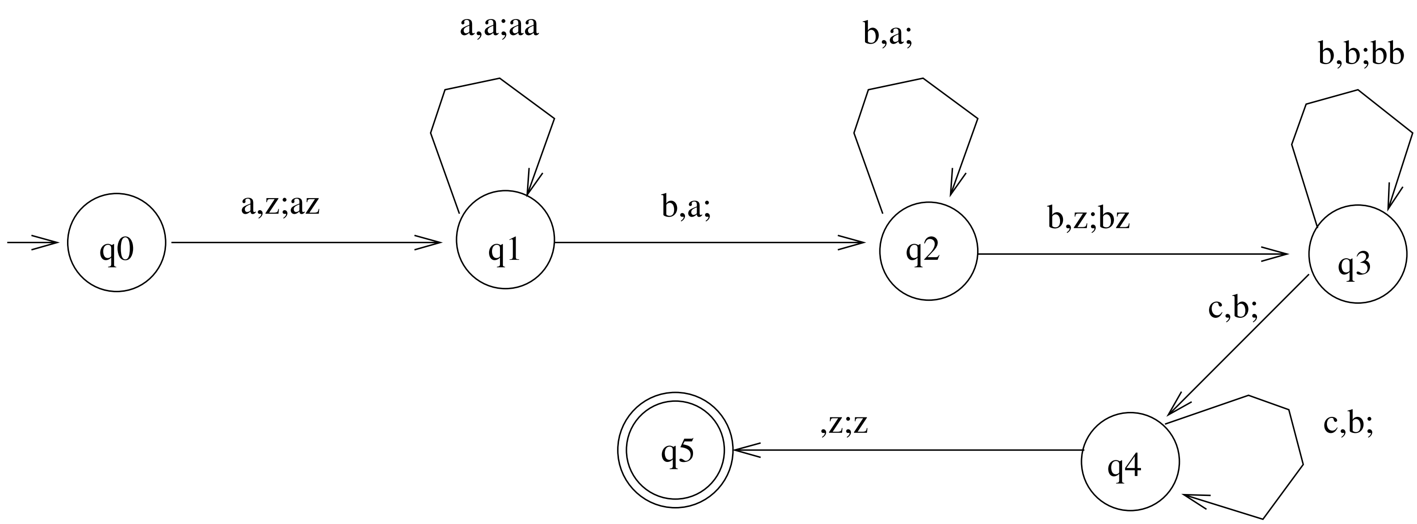
Examples:

1. **Previous pda for** $\{a^n b^n | n \geq 0\}$ **is deterministic?**
2. **Previous pda for** $\{a^n b^m c^{n+m} | n, m > 0\}$ **is deterministic?**
3. **Previous pda for** $\{ww^R | w \in \Sigma^+\}$**,**$\Sigma = \{a, b\}$ **is deterministic?**

Example: L=$\{a^n b^m | m > n, m, n > 0\}$**,**
$\Sigma = \{a, b\}$**,** $\Gamma = \{z, a\}$



Example: L=$\{a^n b^{n+m} c^m | n, m > 0\}$**,**
$\Sigma = \{a, b, c\}$**,**



Example: L=$\{a^n b^{2n} | n > 0\}$**,** $\Sigma = \{a, b\}$