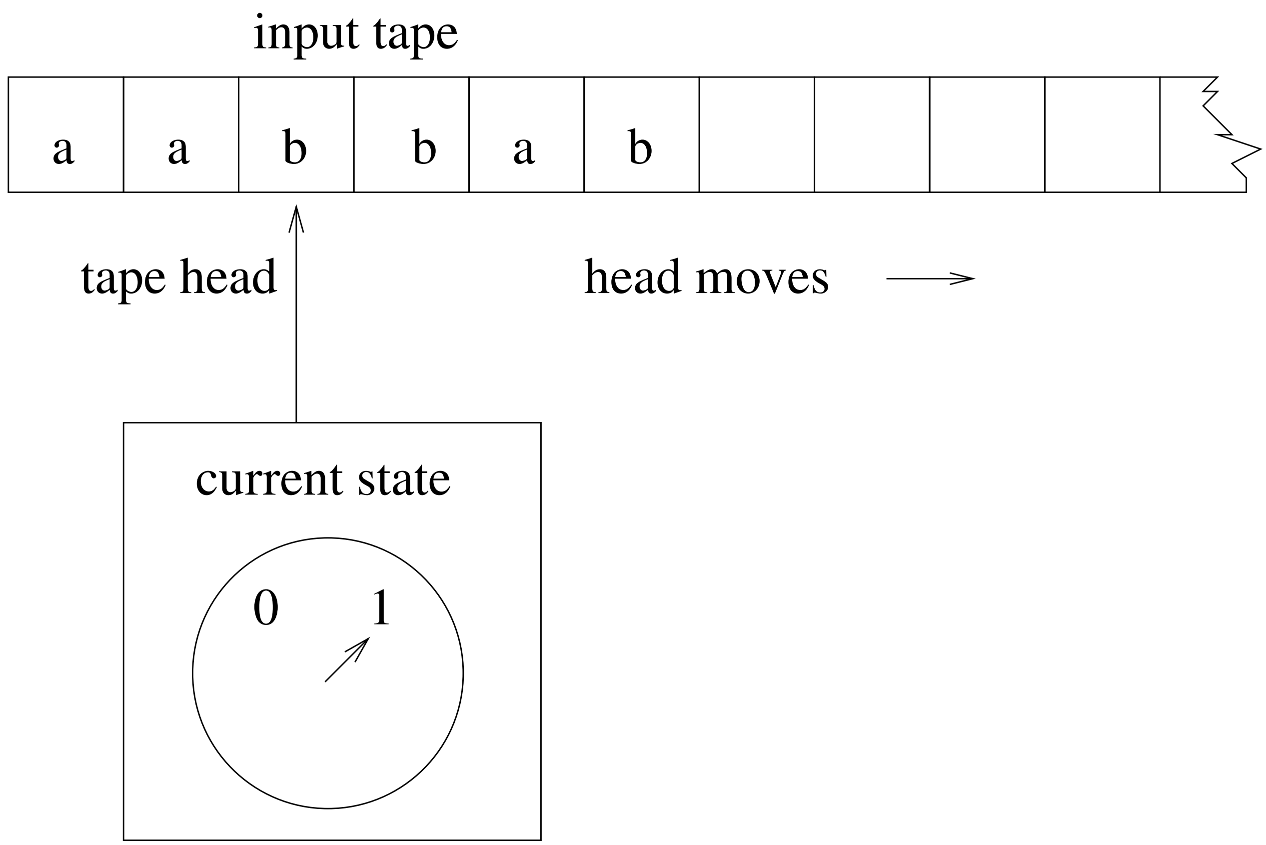
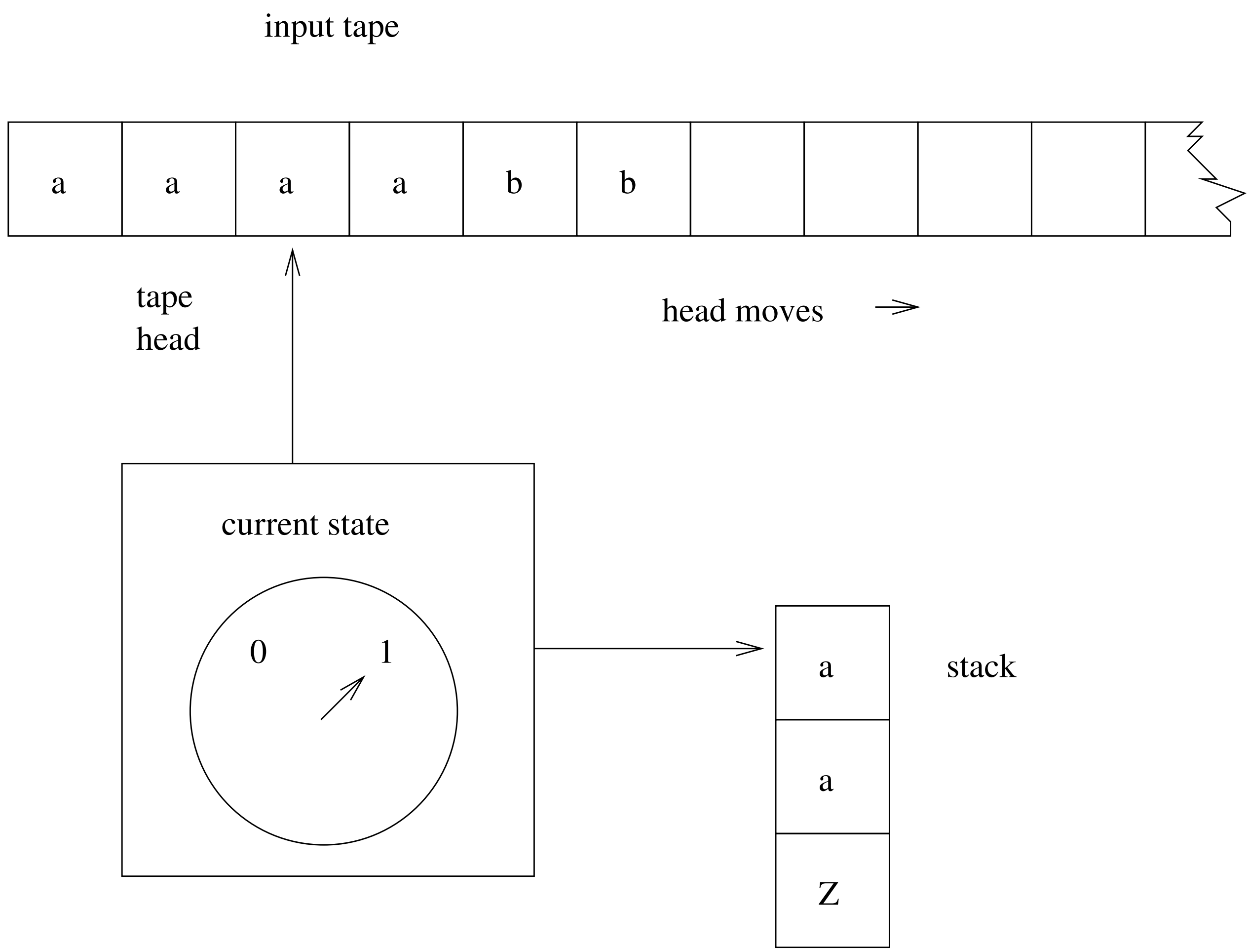
Section: Pushdown Automata

Ch. 7 - Pushdown Automata

A DFA=(Q,Σ,δ,q_0,F)



Modify DFA by adding a stack. New machine is called Pushdown Automata (PDA).



Definition: Nondeterministic PDA (NPDA) is defined by

$$\mathbf{M=(Q,}\Sigma\mathbf{,}\ \Gamma\mathbf{,}\ \delta\mathbf{,}\ q_0\mathbf{,\ z,\ F)}$$

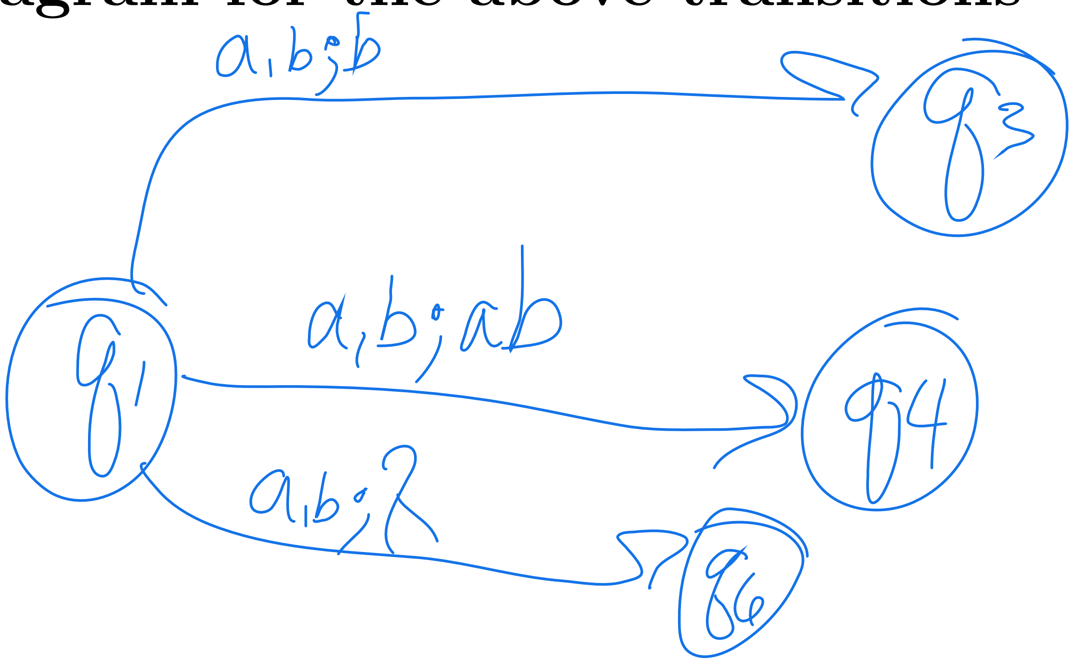
where

Q is finite set of states
Σ **is tape (input) alphabet**
Γ **is stack alphabet**
q_0 **is initial state**
z - start stack symbol, z $\in \Gamma$
F $\subseteq$ **Q is set of final states.**
δ**:Q**$\times(\Sigma \cup \{\lambda\})\times\Gamma\rightarrow$ **finite subsets of** $Q \times \Gamma^*$

Example of transitions

$$\delta(q_1,\mathbf{a},\mathbf{b}) = \{(q_3,\mathbf{b}),(q_4,\mathbf{ab}),(q_6,\lambda)\}$$

The diagram for the above transitions is:



Instantaneous Description:

freeze current state of the machine

$$(\mathbf{q},\mathbf{w},\mathbf{u})$$

on the stack

still to read

Description of a Move:

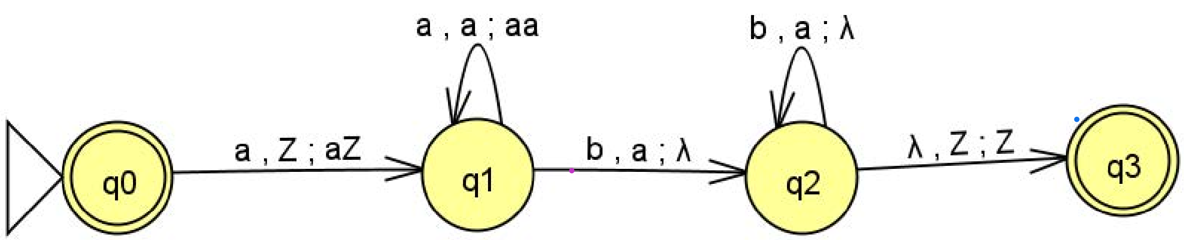
$$(q_1,\mathbf{aw},\mathbf{bx}) \vdash (q_2,\mathbf{w},\mathbf{yx})$$

iff

$$(q_2, y) \in \delta(q_1, a, b)$$

Definition Let $M=(Q,\Sigma,\Gamma,\delta,q_0,z,F)$ be a NPDA. $L(M)=\{w \in \Sigma^* \mid (q_0,w,z) \overset{*}{\vdash} (p,\lambda,u),\ p \in F,\ u \in \Gamma^*\}$. The NPDA accepts all strings that start in q_0 and end in a final state.

Example: L=$\{a^n b^n | n \geq 0\}$**,** $\Sigma = \{a, b\}$**,** $\Gamma = \{z, a\}$

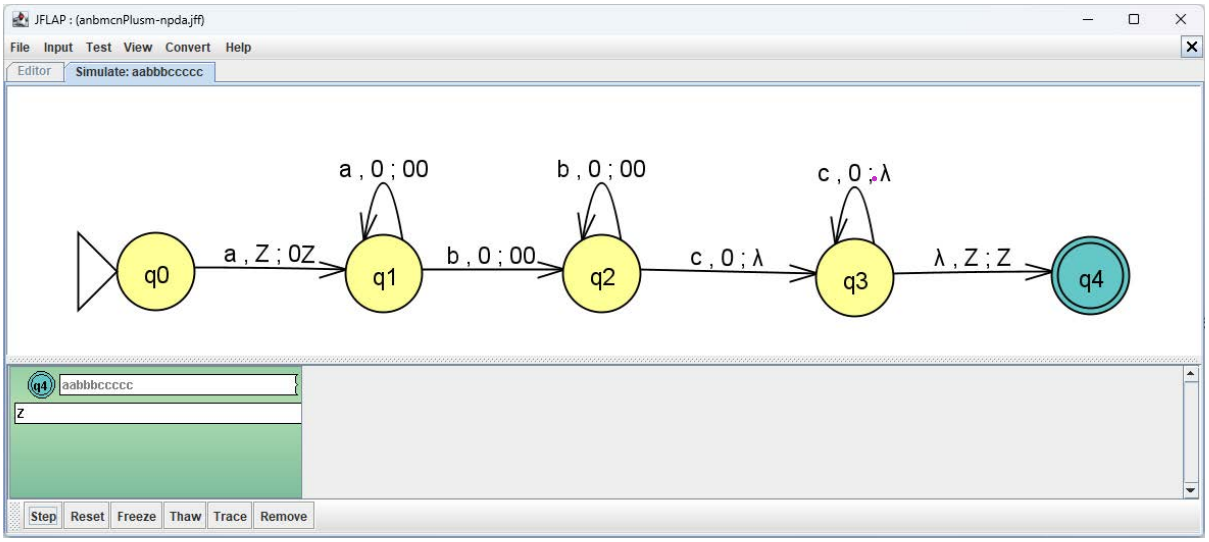


Another Definition for Language Acceptance

NPDA M accepts L(M) by empty stack:

$$\mathbf{L(M)}=\{w \in \Sigma^* | (q_0, w, z) \overset{*}{\vdash} (p, \lambda, \lambda)\}$$

Example: $\mathbf{L}=\{a^n b^m c^{n+m} | n, m > 0\}$, $\Sigma = \{a, b, c\}$, $\Gamma = \{0, z\}$



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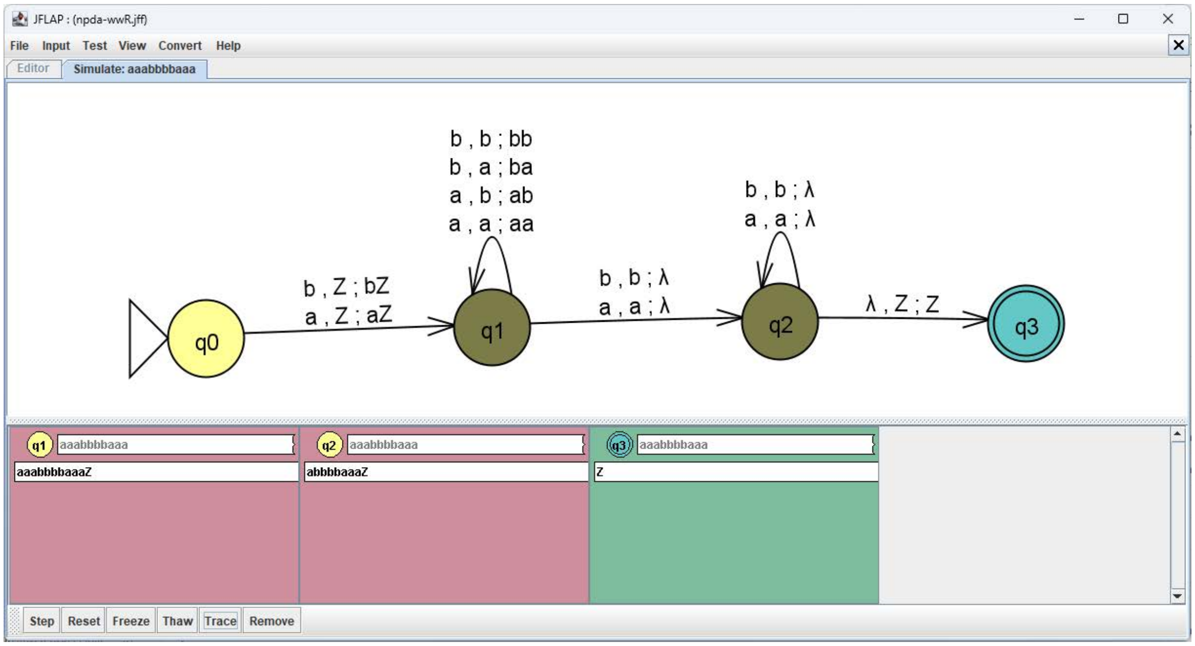
From Classwork last time: ww^R



Examples for you to try on your own: (solutions are at the end of the handout).

- **L=**$\{a^n b^m | m > n, m, n > 0\}$**,** $\Sigma = \{a, b\}$**,** $\Gamma = \{z, a\}$
- **L=**$\{a^n b^{n+m} c^m | n, m > 0\}$**,** $\Sigma = \{a, b, c\}$**,**
- **L=**$\{a^n b^{2n} | n > 0\}$**,** $\Sigma = \{a, b\}$

Definition: A PDA M=(Q,Σ,Γ,δ,q_0,z,F) is *deterministic* **if for every** $q \in$**Q,** $a \in \Sigma \cup \{\lambda\}$, $b \in \Gamma$

1. $\delta(q, a, b)$ **contains at most 1 element**
2. **if** $\delta(q, \lambda, b) \neq \emptyset$ **then** $\delta(q, c, b)$=$\emptyset$ **for all** $c \in \Sigma$

Definition: L is DCFL iff $\exists$ DPDA M s.t. L=L(M).

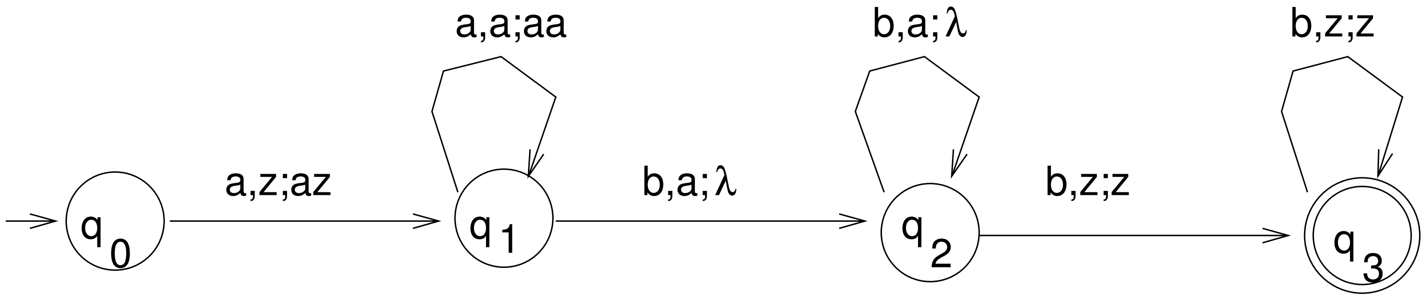
Examples:

1. **Previous pda for** $\{a^n b^n | n \geq 0\}$ **is deterministic?** yes

2. **Previous pda for** $\{a^n b^m c^{n+m} | n, m > 0\}$ **is deterministic?** yes

3. **Previous pda for** $\{ww^R | w \in \Sigma^+\}$**,**$\Sigma = \{a, b\}$ **is deterministic?** No, only nondeterministic

Example: L=$\{a^n b^m | m > n, m, n > 0\}$**,**
$\Sigma = \{a, b\}$**,** $\Gamma = \{z, a\}$



Example: L=$\{a^n b^{n+m} c^m | n, m > 0\}$**,**
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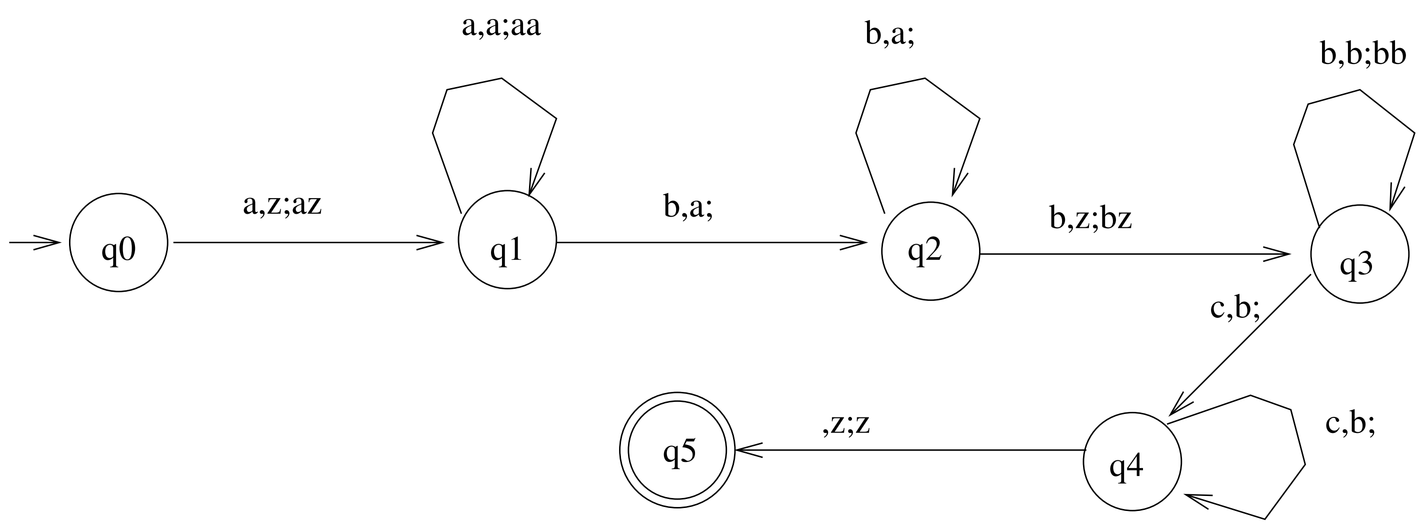

Example: L=$\{a^n b^{2n} | n > 0\}$**,** $\Sigma = \{a, b\}$