Section: Turing Machines

Review

Regular Languages

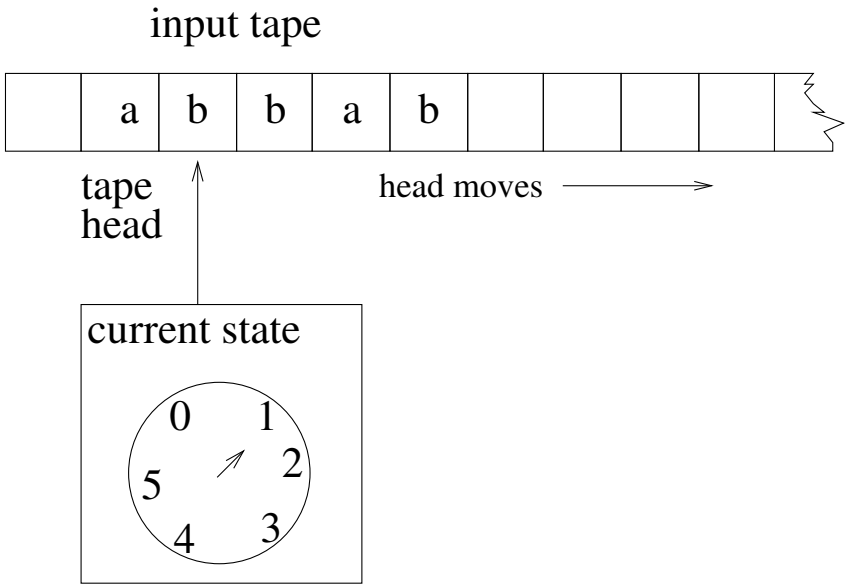
- **FA, RG, RE**
- **recognize** tokens, integer

Context Free Languages

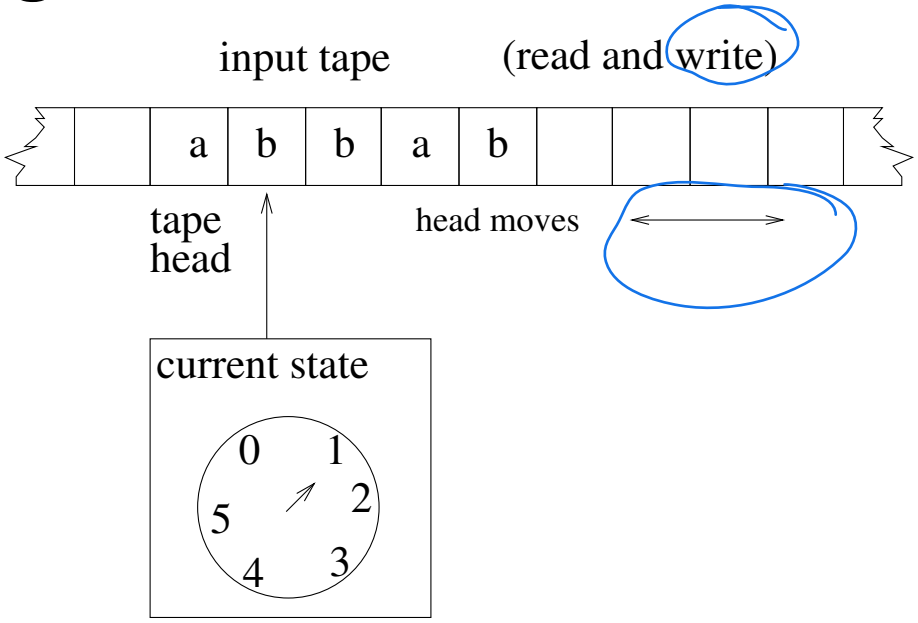
- **PDA, CFG**
- **recognize** arithmitic expression

can't compute exp
can't recognize a^n b^n

DFA:



Turing Machine:



Turing Machine (TM)

- **invented by Alan M. Turing (1936)**
- **computational model to study algorithms**

Definition of TM

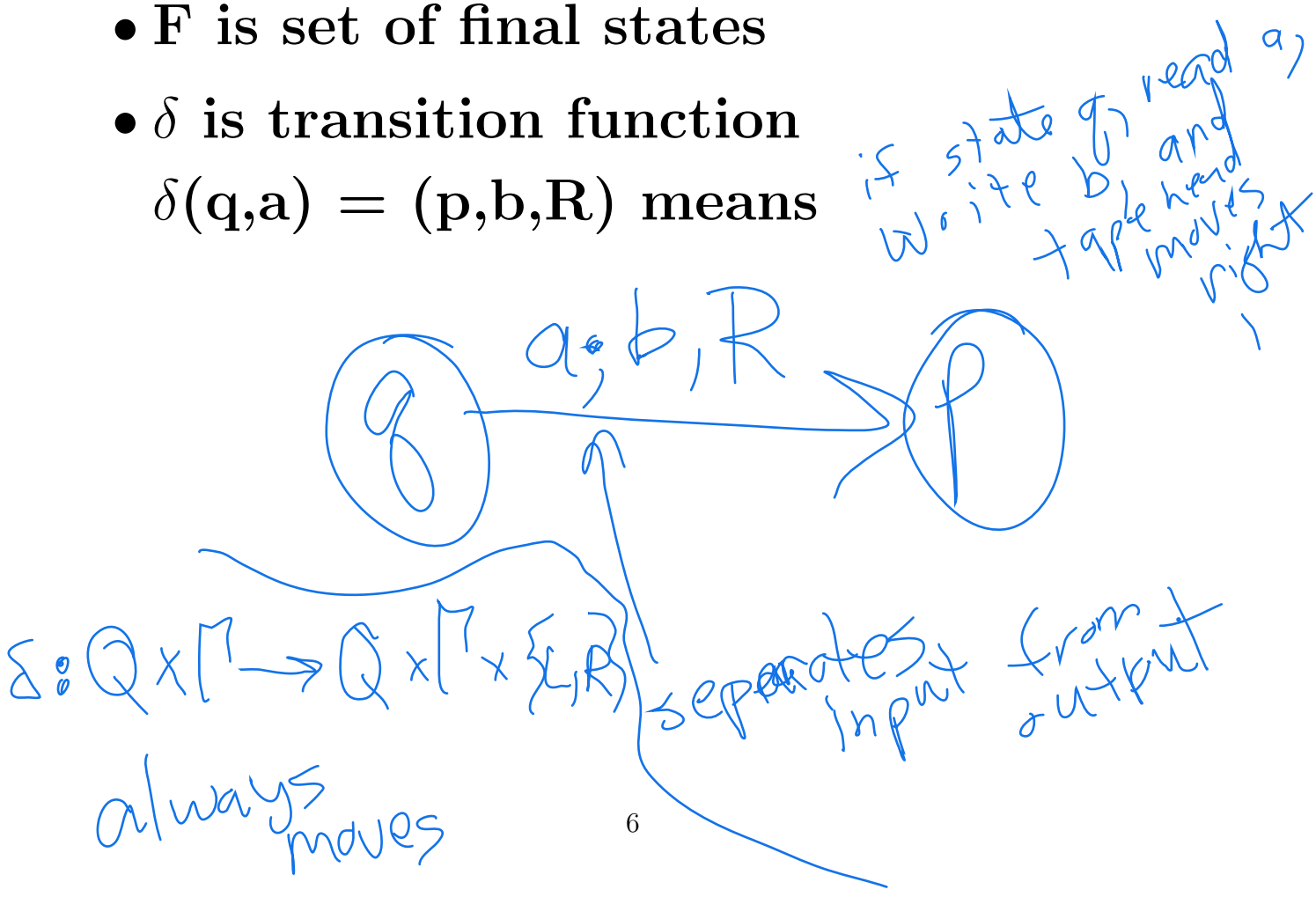
- **Storage**
  - **tape** infinite amount
- **actions**
  - **write symbol**
  - **read symbol**
  - **move left (L) or right (R)**

- **computation**
  - **initial configuration**
    - **start state**
    - **tape head on leftmost tape square**
    - **input string followed by blanks**
  - **processing computation**
    - **move tape head left or right**
    - **read from and write to tape**
  - **computation halts**
    - **final state**

Formal Definition of TM

A TM M is defined by
M=(Q,Σ,Γ,δ,q_0,B,F) where

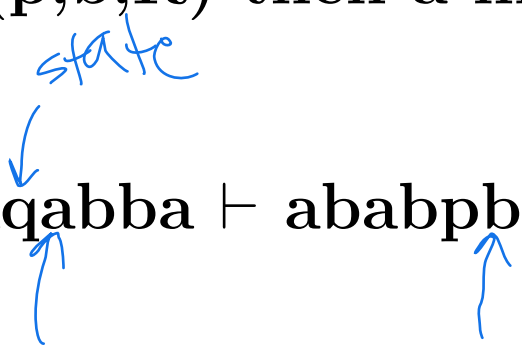
- **Q is finite set of states**
- **Σ is input alphabet**
- **Γ is tape alphabet**
- **B$\in \Gamma$ is blank**
- **q_0 is start state**
- **F is set of final states**
- **δ is transition function**
  δ(q,a) = (p,b,R) means



TM as Language recognizer

Definition: Configuration is denoted by $\vdash$.

if δ(q,a) = (p,b,R) then a move is denoted



Definition: Let M be a TM, M=(Q,Σ,Γ,δ,q_0,B,F). L(M) =
$\{w \in \Sigma^* | q_0 w \overset{*}{\vdash} x_1 q_f x_2$ **for some** $q_f \in$**F,** $x_1, x_2 \in \Gamma^*\}$

TM as language acceptor

M is a TM, w is in Σ^*,

- **if w$\in$ L(M) then M halts in final state**
- **if w$\notin$ L(M) then either**
  - **M halts in non-final state**
  - **M doesn't halt**

TM as a transducer

TM can implement a function: f(w)=w'

start with: **w**

↑

end with: **w'**

↑

Definition: A function with domain D is *Turing-computable* **or** *computable* **if there exists TM M=(Q,**$\Sigma, \Gamma, \delta, q_0$**,B,F) such that**

$$q_0 w \overset{*}{\vdash} q_f f(w)$$

$q_f \in$**F, for all** $w \in$**D.**

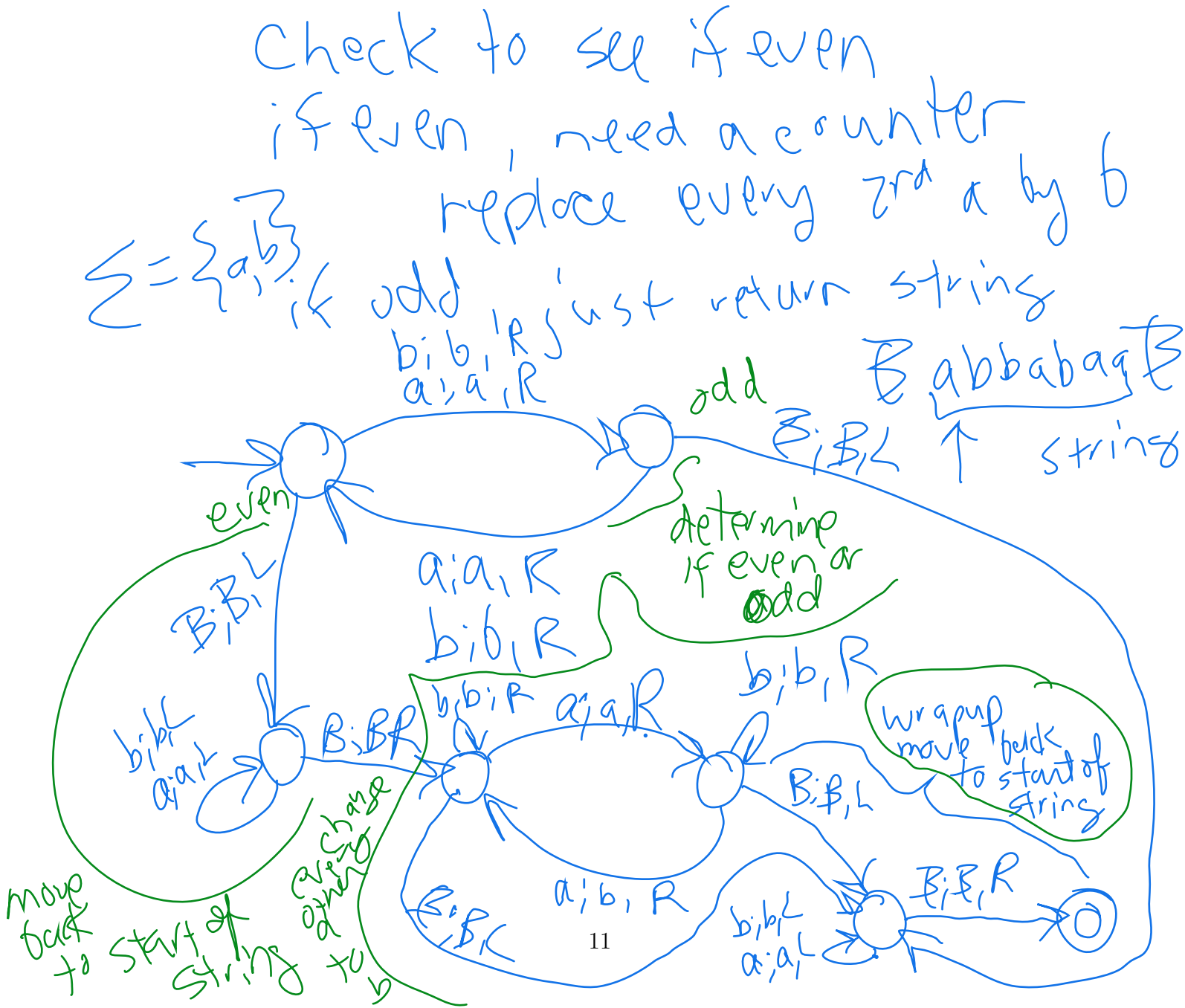
transducer example

Example

$\Sigma = \{a, b\}$

Replace every second 'a' by a 'b' if string is even length.

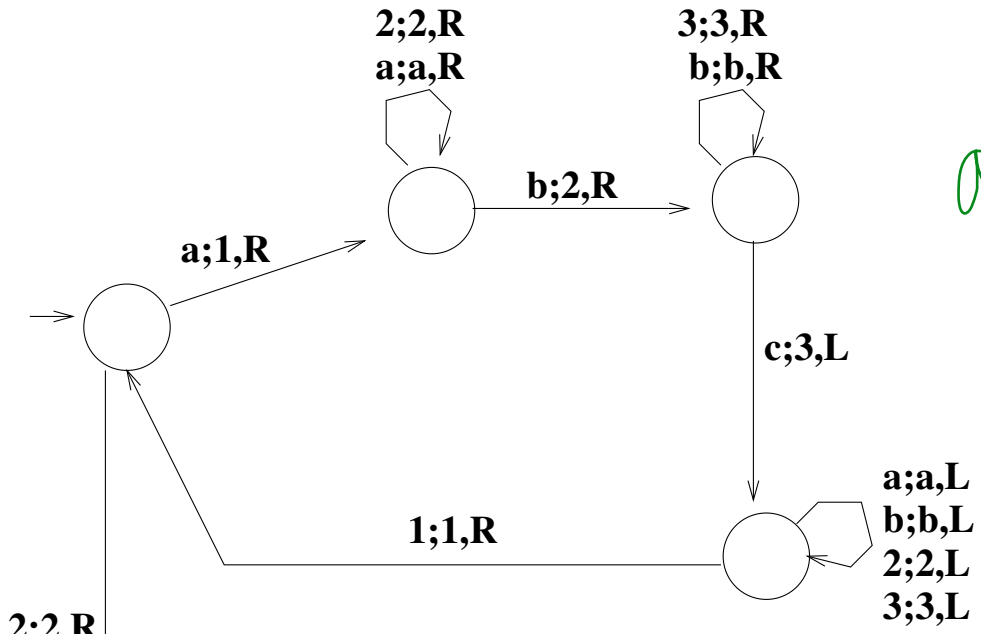
- **Algorithm**

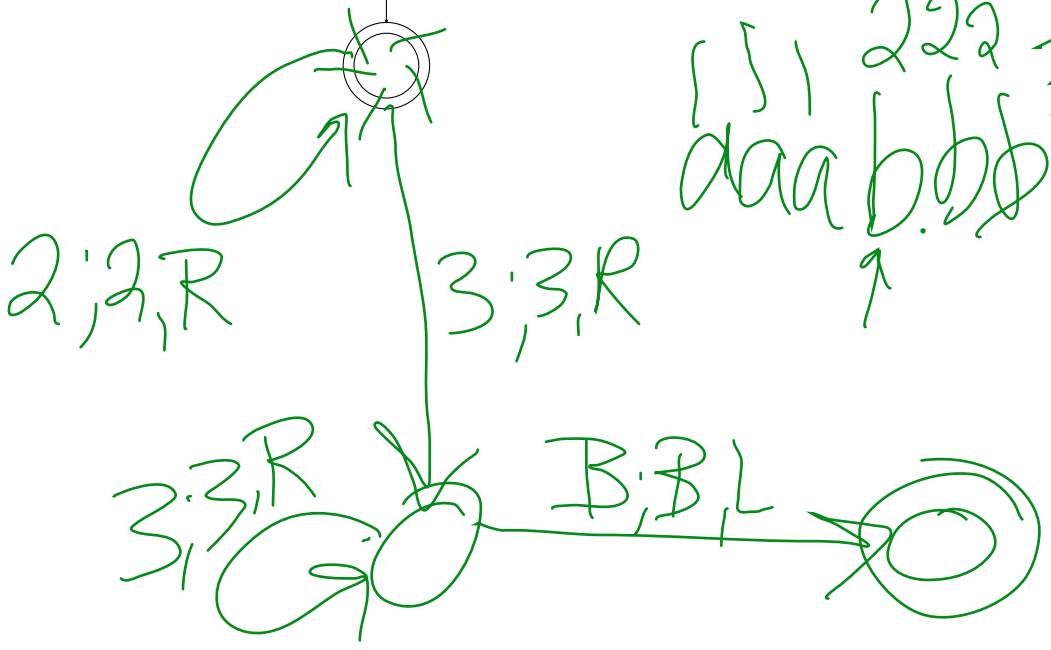


Example:

$\mathbf{L}=\{a^n b^n c^n | n \geq 1\}$

Is the following TM Correct?





example Language acceptor

aabbccc ∉ L but accepts

111 222 333 3
aaabbbccc ∈ L
works

now aabbccc not accepted

transducer

Example:

f(x) = 2x

x is a unary number

start with:	**111**
	↑
end with:	**111111**
	↑

Is the following TM correct?