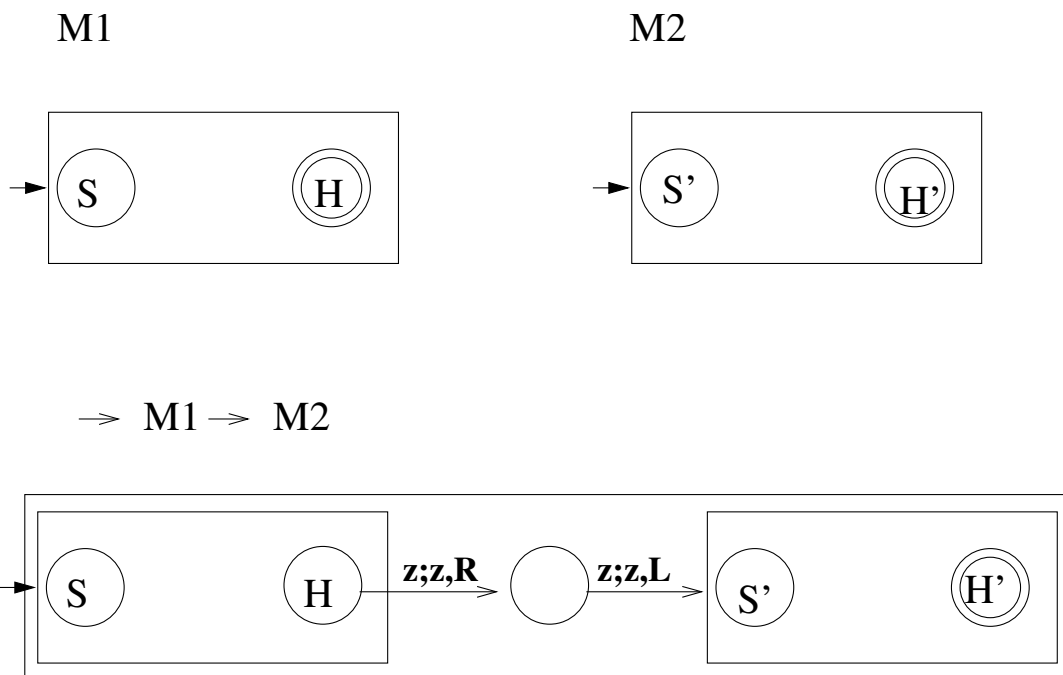


Section: Turing Machines - Building Blocks

1. Given Turing Machines M1 and M2 Notation for

- Run M1
- Run M2

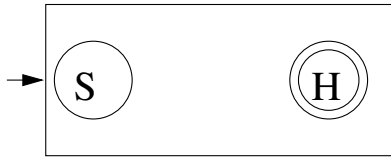


z represents any symbol in

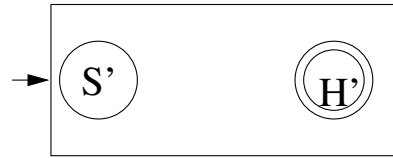


2. Given Turing Machines M1 and M2

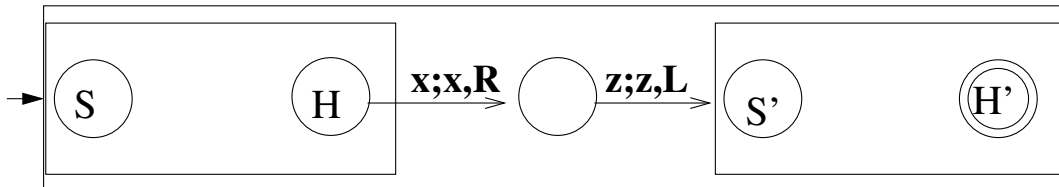
M1



M2

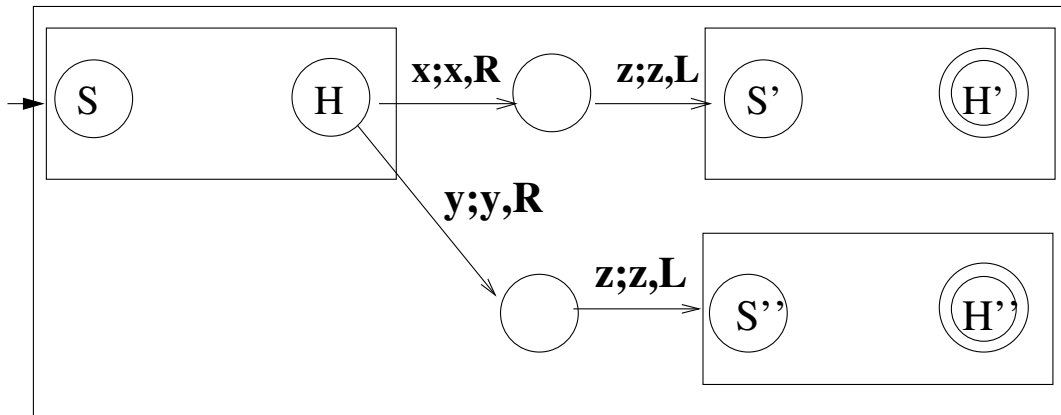
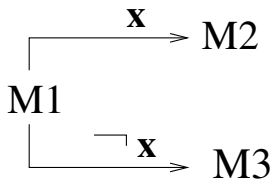
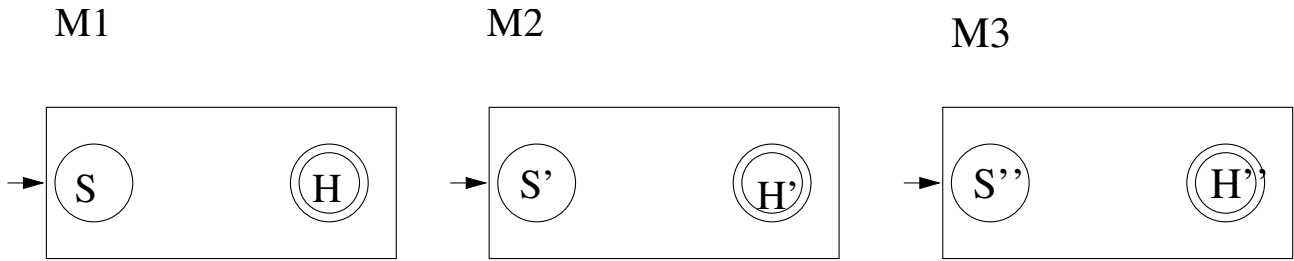


$\Rightarrow M1 \xrightarrow{x} M2$



z represents any symbol in
x is an element of

3. Given Turing Machines M1, M2, and M3



x is an element of Σ
 y is any element except x from Σ
 z is any element from Σ

More Notation for Simplifying Turing Machines

Suppose $\Gamma = \{a, b, c, B\}$

z is any symbol in Γ

x is a specific symbol from Γ

1. s - start

2. R - move right

3. L - move left

4. x - write x (and don't move)

5. R_a - move right until you see an a

6. L_a - move left until you see an a

7. $R_{\neg a}$ - move right until you see anything that is not an a

8. $L_{\neg a}$ - move left until you see anything that is not an a

9. h - halt in a final state

10. $\left. \begin{array}{l} a, b \\ \rightarrow \end{array} \right\} \rightarrow w$

If the current symbol is a or b , let w represent the current symbol.

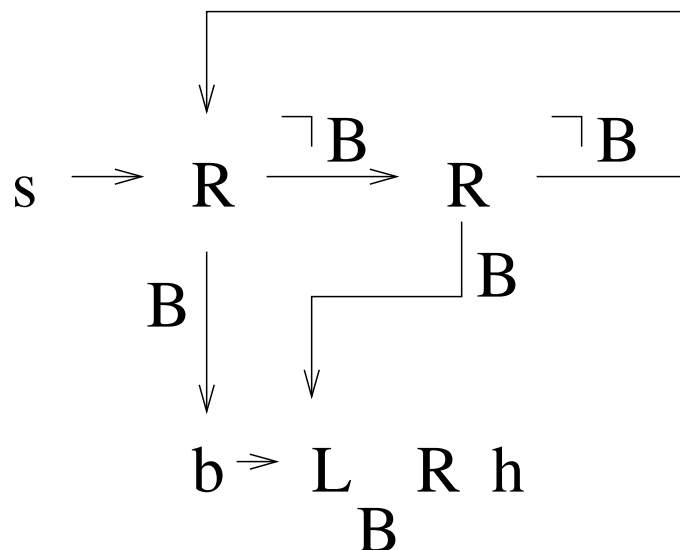
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$.

If $|w|$ is odd, then write a b at the end of the string. The tape head should finish pointing at the leftmost symbol of w .

input: bab , output: $babb$

input: ba , output: ba



What is the running time?

Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$,
 $|w| > 0$

For each a in the string, append a b to the end of the string.

input: *abbabb*, output: *abbabbbb*

The tape head should finish pointing at the leftmost symbol of w .

Turing's Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An *algorithm* for a function $f:D \rightarrow R$ is a TM M , which given input $d \in D$, halts with answer $f(d) \in R$.

Example: $f(x + y) = x + y$, x and y unary numbers.

start with: 111+1111

↑

end with: 1111111

↑

Example: Copy a String, $f(w) = w0w$,
 $w \in \Sigma^*$, $\Sigma = \{a, b, c\}$

Denoted by C

start with: abac

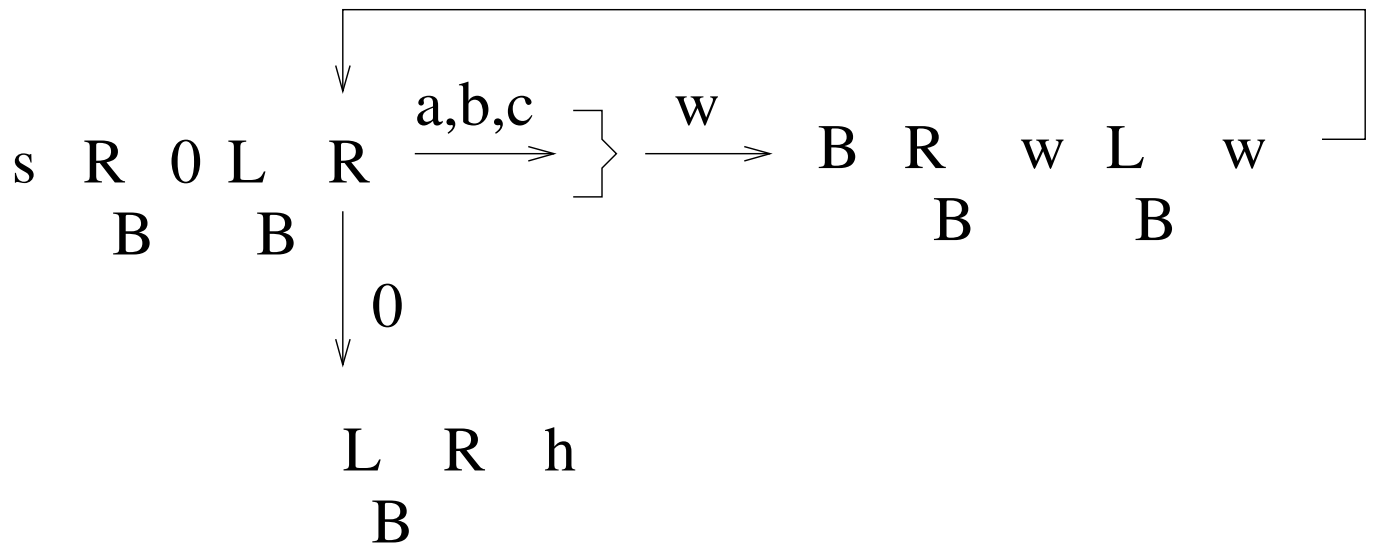
↑

end with: abac0abac

↑

Algorithm:

- **Write a 0 at end of string**
- **For each symbol in string**
 - **make a copy of the symbol**



Example: Shift the string that is to the left of the tape head to the right, denoted by S_R (shift right)

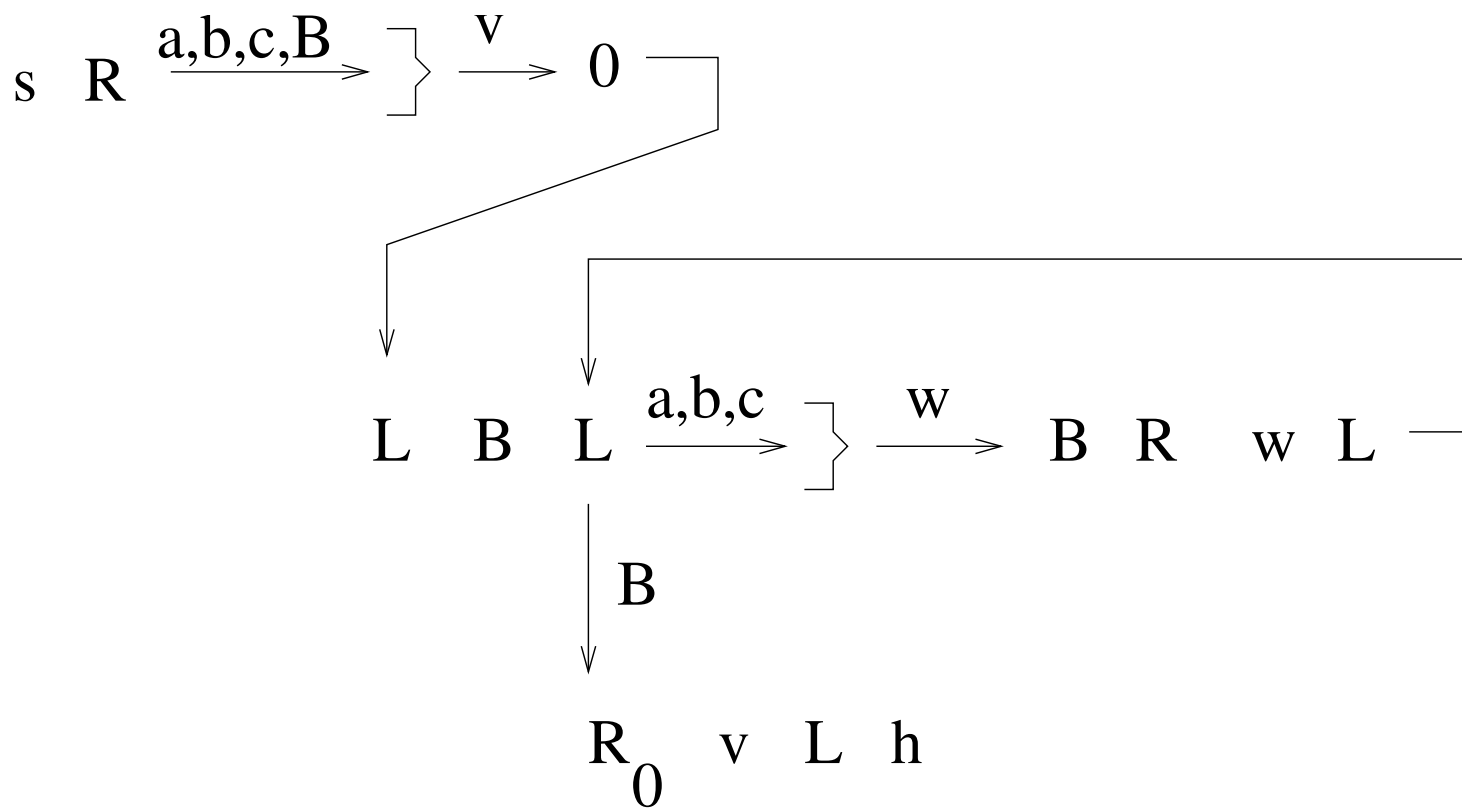
Below, “ba” is to the left of the tape head, so shift “ba” to the right.

start with: aaBbabca
 ↑

end with: aaBBbaca
 ↑

Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
 - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position



Example: Shift the string that is to the right of tape head to the left, denote by S_L (shift left)

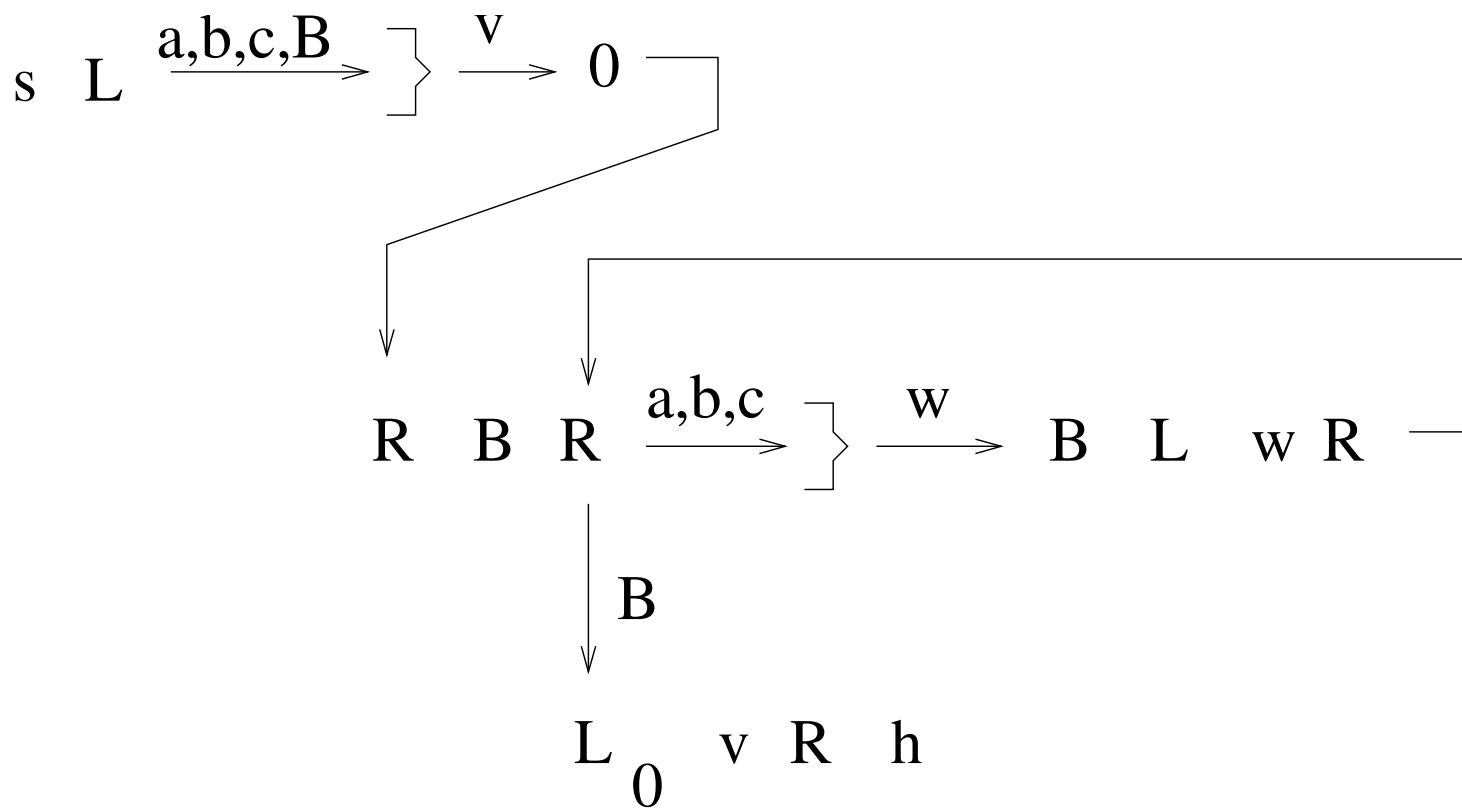
start with: **babcaBba**

↑

end with: **ba**c**aBBba**

↑

(similar to S_R)



Example: Add unary numbers

This time use shift.

Example: Multiply two unary numbers, $f(x*y)=x*y$, x and y unary numbers. Assume $x,y>0$.

start with: 1111*11
 ↑

end with: 11111111
 ↑