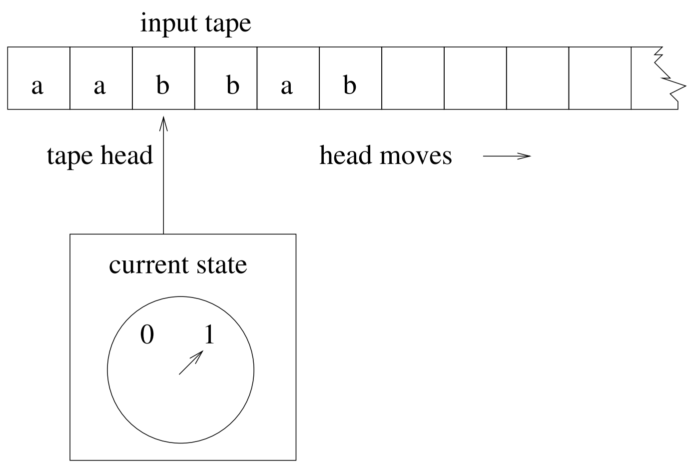
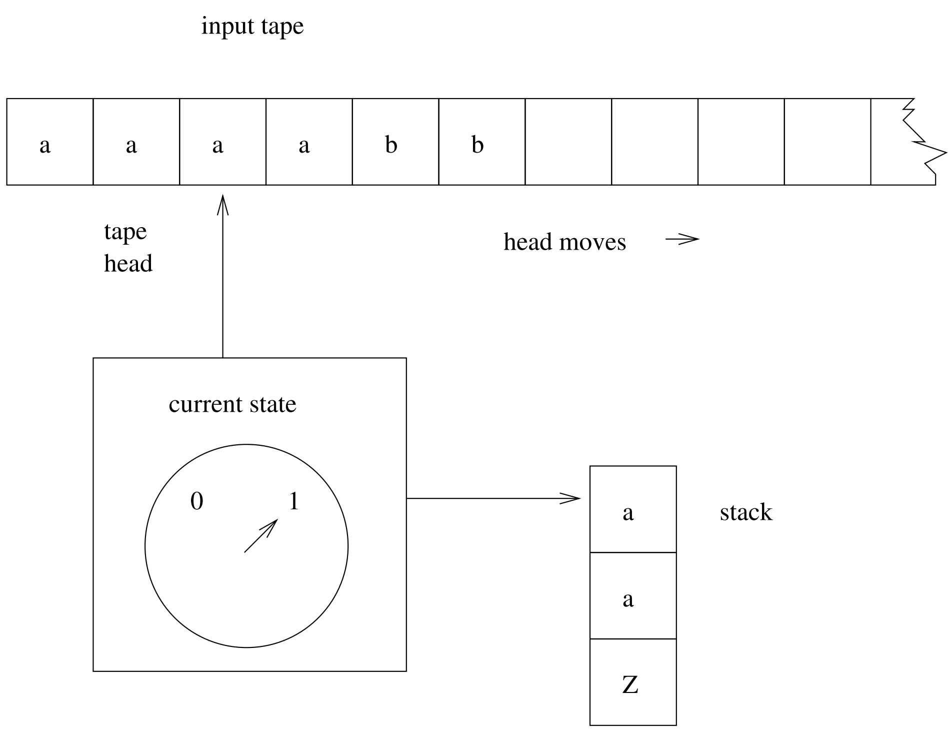
Compsci 334 - Mathematical Foundations of CS
Dr. S. Rodger
Section: Pushdown Automata (Ch. 7) (handout)

Ch. 7 - Pushdown Automata

A DFA=(Q,Σ,δ,q_0,F)



Modify DFA by adding a stack. New machine is called Pushdown Automata (PDA).



Definition: Nondeterministic PDA (NPDA) is defined by

$$M=(Q,\Sigma, \Gamma, \delta, q_0, z, F)$$

where

Q is finite set of states
Σ is tape (input) alphabet
Γ is stack alphabet
q_0 is initial state
z - start stack symbol, (bottom of stack marker), z $\in \Gamma$
F $\subseteq$ Q is set of final states.
δ:Q$\times(\Sigma \cup \{\lambda\})\times\Gamma\rightarrow$ finite subsets of $Q \times \Gamma^*$

Example of transitions

$\delta(q_1$,a,b) = $\{(q_3$,b),$(q_4$,ab),$(q_6,\lambda)\}$

Meaning: If in state q_1 with "a" the current tape symbol and "b" the symbol on top of the stack, then pop "b", and either

move to q_3 and push "b" on stack
move to q_4 and push "ab" on stack ("a" on top)
move to q_6

Transitions can be represented using a transition diagram.

The diagram for the above transitions is:

Each arc is labeled by a triple: x,y;z where x is the current input symbol, y is the top of stack symbol which is popped from the stack, and z is a string that is pushed onto the stack.

Instantaneous Description:

(q,w,u)

Notation to describe the current state of the machine (q), unread portion of the input string (w), and the current contents of the stack (u).

Description of a Move:

$$(q_1,\text{aw},\text{bx}) \vdash (q_2,\text{w},\text{yx})$$
iff

Definition Let M=(Q,Σ,Γ,δ,q_0,z,F) be a NPDA. L(M)=$\{$w$\in \Sigma^*$ | (q_0,w,z) $\overset{*}{\vdash}$ (p,λ,u), p$\in$F, u$\in \Gamma^*\}$. The NPDA accepts all strings that start in q_0 and end in a final state.

Example: L=$\{a^n b^n | n \geq 0\}$, $\Sigma = \{a, b\}$, $\Gamma = \{z, a\}$

Another Definition for Language Acceptance

NPDA M accepts L(M) by empty stack:

$$\text{L(M)}=\{w \in \Sigma^* | (q_0, w, z) \overset{*}{\vdash} (p, \lambda, \lambda)\}$$

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Example: L=$\{a^n b^m c^{n+m} | n, m > 0\}$, $\Sigma = \{a, b, c\}$, $\Gamma = \{0, z\}$

Examples for you to try on your own: (solutions are at the end of the handout).

- L=$\{a^n b^m | m > n, m, n > 0\}$, $\Sigma = \{a, b\}$, $\Gamma = \{z, a\}$
- L=$\{a^n b^{n+m} c^m | n, m > 0\}$, $\Sigma = \{a, b, c\}$,
- L=$\{a^n b^{2n} | n > 0\}$, $\Sigma = \{a, b\}$

Definition: A PDA M=(Q,Σ,Γ,δ,q_0,z,F) is *deterministic* if for every $q \in$Q, $a \in \Sigma \cup \{\lambda\}$, $b \in \Gamma$

1. $\delta(q, a, b)$ contains at most 1 element
2. if $\delta(q, \lambda, b) \neq \emptyset$ then $\delta(q, c, b)$=$\emptyset$ for all $c \in \Sigma$

Definition: L is DCFL iff $\exists$ DPDA M s.t. L=L(M).

Examples:

1. Previous pda for $\{a^n b^n | n \geq 0\}$ is deterministic?
2. Previous pda for $\{a^n b^m c^{n+m} | n, m > 0\}$ is deterministic?
3. Previous pda for $\{ww^R | w \in \Sigma^+\}$,$\Sigma = \{a, b\}$ is deterministic?