

Roadmap

- Rank aggregation: merging ranked results from different searches
 - Fagin et al. "Optimal Aggregation Algorithm for Middleware." PODS, 2001
- Proximity search: finding all shortest paths in the link structure of a database
- Goldman et al. "Proximity Search in Databases." VLDB, 1998
- WSQ: enhancing database queries with Web searches
 - Goldman and Widom. "WSQ/DSQ: A Practical Approach for Combined Querying of Databases and the Web." *SIGMOD*, 2000

Rank aggregation problem

- Each object *R* is graded using *m* different criteria
 - Grades are $x_1, x_2, ..., x_m$
 - Example: each grade measures the relevance of R for a particular search term
- The combined grade is calculated by an aggregation function $t(x_1, x_2, ..., x_m)$
 - Example: weighted sum, min, max, etc.
- Problem: find the top *k* objects with the highest combined grade

Modes of access

- Sorted: sequentially request a list of objects (with their grades) ranked according to one criterion (from highest to lowest)
 - Conceptually, there are *m* lists $L_1, L_2, ..., L_m$, one for each criterion
 - Example: search an image database for a list of "red" pictures, ranked by their "redness"
- Random: request the grade of an object according to one criterion
 - Example: query an image database for the "redness" of one particular picture

Assumptions

- Individual scores are in the interval [0, 1]
- Aggregation function is monotone
 - If $x_i \le x'_i$ for every *i*, then $t(x_1, x_2, ..., x_m) \le t(x'_1, x'_2, ..., x'_m)$
 - Examples: min, sum
 - Not an example: $1 \operatorname{sum}(x_1, x_2, \ldots, x_m)$
- Cost per sorted access (one object, one score): c_S
- Cost per random access (one object, one score): c_R

Naïve algorithm

- For each criterion, do sorted access to retrieve all objects and their grades
 - That is, access all L_i 's
- Calculate the combined grades for all objects
- Pick the top k
- Accesses the entire database
- \triangleright Does not use the fact that L_i 's are sorted

FA (Fagin's Algorithm)

- Do sorted access in parallel to all *L*_{*i*}'s, until there are *k* "matches"
 - A "match" is an object that has been seen in all L_i 's
- For each object that has been seen, do random accesses to get all its grades
- Calculate the combined grades and pick the top k
- ≻ Pop quiz
 - Why not just consider the k matches?
 - Why not consider objects seen after the k matches?
- ➤ Needs to remember lots of objects: large buffer size
- Does not use the aggregation function effectively

TA (Threshold Algorithm)

- Do sorted access in parallel to all L_i 's
 - Whenever an object is seen, do random accesses to get all its grades, and compute the combined grade
 - Remember up to k objects with top combined grades
 - Calculate the threshold value $\tau = t(\underline{x}_1, \underline{x}_2, ..., \underline{x}_m)$, where \underline{x}_i 's are the bottom grades seen so far
 - If we have seen at least k objects whose combined grade is at least τ , stop
- Output the top k objects we remembered

Intuition behind TA

- τ serves as an upper bound on the combined grade for objects that have never been seen
- When we stop, the top *k* objects we have remembered all have combined grade of at least *τ*
- > The top k we have are the top k overall
- "Gather what information you need to allow you to know the top *k* answers, and then halt"

FA versus TA

- TA never stops later than FA
 - FA's stopping condition (k matches) implies that TA's stopping condition (k objects above threshold) has already been satisfied
- TA requires only bounded buffers (to remember the top *k* objects)
- TA may perform more random accesses

Instance optimality

- A: a class of algorithms
- D: a class of legal inputs to algorithms
- *cost*(*A*, *D*): cost of running algorithm *A* with input *D*
- An algorithm $B \in \mathbf{A}$ is instance optimal over \mathbf{A} and \mathbf{D} if for every $A \in \mathbf{A}$ and every $D \in \mathbf{D}$, cost(B, D) = O(cost(A, D))
 - That is, $cost(B, D) \le c \cdot cost(A, D) + c'$, where c is called the optimality ratio

> Much stronger than worst-case optimality

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Instance optimality of TA

- A: the class of all algorithms that do not make mistakes or wild guesses
 - "No wild guesses" means no random access for *R* unless *R* has been seen in sorted access
- D: the class of all possible inputs
- TA is instance optimal over A and D, with optimality ratio of at most $m + m(m-1) c_R/c_S$

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Intuition behind TA's instance optimality

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Seen 1

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- Say an algorithm A stops sooner than TA on some input (i.e., one of the top k picked by A has combined grade less than τ)
- Construct another input by inserting a new object *R* right below where *A* stops looking
- Then *A* will make an mistake by failing to pick *R*
- See paper for a rigorous proof and other results

Related work: Quick-Combine

- Güntzer et al. "Optimizing Multi-Feature Queries in Image Databases." VLDB, 2000
- Instead of doing sorted access on all Li's in parallel, choose one L_i to access next
- Prefer the list in which grades are declining at the fastest rate, so we can lower the threshold value faster
 Rate is measured by the decrease in successive grades,
 - weighted by $|\partial t/\partial x_i|$
- $\geq |\partial t / \partial x_i|$ may be undefined (e.g., min)
- Quick-Combine may beat TA in some cases, but is not instance optimal unless we ensure every L_i is accessed every now and then

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Extending TA

- What if we only need approximate answers? TA_θ
 Example: Web search, with lots of good-quality answers
- What if we have no sorted access for some criteria? TA_Z
 Example: find good restaurants near me (sorted and random accesses for restaurant ratings, random access only for distances)
- What if we have no random access at all? NRA
 Example: Web search engines, which typically do not allow you to enter a URL and get its ranking
- What if consider the relative costs of random and sorted accesses? CA

Adding approximation: TA_{θ}

- A θ -approximation ($\theta > 1$) to the top k answers is a collection of k objects, such that
 - For each *R* among these *k* objects, and for each *R*' not among these these *k* objects, $\theta t(R) \ge t(R')$
- TA_{θ}: same as TA, except that the stopping condition is "we have seen at least *k* objects whose combined grade is at least τ/θ "

Restricting sorted access: TA_Z Suppose we only have sorted access to L_i for i∈Z TA_Z: same as TA, except Only do sorted accesses to L_i's where i∈Z Use x_j = 1 to calculate the threshold if sorted access to L_i is not allowed (j∉Z)

> Intuition: The first object that we see in L_j (if we could) can have grade 1

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NRA

- Do sorted access in parallel to all L_i 's; at each depth:
 - Maintain bottom grades $\underline{x}_1, \underline{x}_2, \dots, \underline{x}_m$ seen in the lists
 - For each object *R*, calculate the lower and upper bounds for its combined grade
 - If x_i is not available, use 0 in calculating lower bound, and use \underline{x}_i in calculating upper bound
 - Maintain a current top k list containing the k objects with the largest lower bounds (ties are broken by upper bounds)
- · Stopping condition
 - For every R not in the current top k list, the calculated upper bound for R is less than the calculated lower bound for all objects in the current top k list
- Lots of note-keeping!

Combined algorithm: CA

- A compromise between TA (lots of random accesses) and NRA (no random accesses)
 - Uses random accesses, but considers their cost relative to sorted access
- Suppose $h = \lfloor c_R / c_S \rfloor \ge 1$
- CA: same as NRA, except
 - Every h steps, pick an object with missing individual grades (see paper for details on which object to pick)
 - Do random accesses to get the missing grades, and then recalculate the bounds

Summary

- A very practical problem (rank aggregation)
- A simple algorithm (TA) and various extensions
 - Nothing wild; just clean implementations of remarkably simple ideas (thresholds, bounds)
 - But the amazing thing is that these simple algorithms are unbeatable for any input (instant optimality)!

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