Part I. Review: Pre-calculus, Calculus & Linear Algebra

1. Google's PageRank.

   Read the paper by Sergey Brin and Lawrence Page titled *The PageRank Citation Ranking: Bringing Order to the Web.*

   (a) (5) List the technical terms you were not familiar with before reading the paper.

   (b) (5) List the new technical terms you understand while reading the paper and some other references.

   (c) (10) Give a few comments on the ideas described in the paper. Cite other related papers you have read.

2. POLYNOMIALS.

   Let $k \geq 0$ be an integer. Denote by $P_k(a, b)$ the set of all polynomials of degree $\leq k$ on the interval (a, b).

   (a) (3) How many elements in $P_0(a, b)$?
   How many elements in $P_{1234567}(a, b)$?
   How many elements in $P_n(a, b)$ for $n > 0$?

   (b) (2) How many points are on the curve corresponding to a polynomial $p \in P_1(0, 2)$?

   (c) (8) Show that any real polynomial of degree $k > 0$ is unbound over $(-\infty, \infty)$.

   (d) (5) Let p be a polynomial of degree $< n$. Show that $p^{(n)}(z) = 0$.

   (e) (6) Prove that $\cos(x)$ is not a polynomial.

   (f) (6) Show that $P_k(a, b)$ is a vector space on the complex field, with the conventional pointwise addition and multiplication by scalar. Find the dimension of the space.

   (g) (5) Comment on comparison between Weierstrass theorem and Taylor's theorem on function approximation with polynomials.

Optional. (5) Find out what Littlewood's conjecture is.

Optional. (5) List some basic properties of trigonometric polynomials.

Part II. Use of MATLAB and Taylor polynomials

1. (30) Write a MATLAB **function** to efficiently evaluate $\cos(x)$ at a given evaluation point x within a given accuracy tolerance. Error estimation must be included.

   Hint: use scaling, translation, partition and *seeds*, approximation with Taylor polynomials, and Horner's rule.

2. (15) Write a MATLAB **script** to plot the evaluations at $[0:100]*\pi/100$, the operations used for each point, and the approximation error at each point. In other words, show the function curve at the sample points and demonstrate the computation complexity and accuracy.