

## Part I.

## 1. DFT-related special matrices.

- (a) (5) Find out connections between Toeplitz matrices, Hankel matrices, and circulant matrices.
- (b) (10) The (discrete) convolution of two sequences  $x(i)$  and  $h(i)$  is a sequence  $y(j)$  defined as follows

$$y(j) = \sum_{i=0}^{\infty} x(j-i)h(i), \quad 0 \leq j < \infty.$$

The convolution is well defined if the summation converges for every  $j$ . In particular, it is well defined when  $x$  and  $h$  are finite. In signal processing,  $x(j-i)$  is the time-reversed signal and  $h(i)$  is the filter.

Describe the problem in matrix form and give a fast convolution algorithm.

- (c) (10) The (discrete) correlation of two sequences  $a(i)$   $b(i)$  is a sequence

$$c(j) = \sum_{i=-\infty}^{+\infty} a(i+j)b(i), \quad -\infty \leq j < +\infty.$$

It is used for comparison of two sequences or signals with all possible shifts  $j$ .

Describe the problem in matrix form and give a fast convolution algorithm.

2. (20) Algorithm designs for the first two MATLAB prototypes.
3. (10) Describe the google matrix in the PageRank paper.

## Part II. MATLAB EXPERIMENTS.

1. (10) Provide a function for the two rings comparison problem, based on FFT algorithm.
2. (10) Provide a function for fast multiplication of two polynomials, based on FFT algorithm.  
**Optional.** (5) Provide better solutions to the above problems.
3. (5) Provide a function for LS curve fitting with weights.
4. (10) Audio compression.
5. (10) Image restoration.