METHODS FOR THE SOLUTION OF LINEAR EQUATION SYSTEMS

Part I.

1. Discrete Poisson Equation with Finite Difference. See Equation (12.1) and Equation (12.4) in the text book.

   (a) The one-dimensional case. Consider the case that

   $$f(x,y) = -y\cos(x), \quad \partial u/\partial y = g(x), \quad (x,y) \in [0,\pi] \times [0,1].$$

   for some function g. The boundary conditions are

   $$u(x,0) = 0, \quad u(x,1) = \cos(x), \quad u(0,y) = y, \quad u(\pi,y) = -y.$$

   Let $h = \pi/n$ and $k = 1/m$ with integers n and m. Let $x_i = ih$ and $y_j = jk$.

   (b) The two-dimensional case. See Problem 8 in Exercise set 12.1.

   (c) **optional.** The three-dimensional case. Extend the two dimensional case.

   For each case,

   - Let $m = n = 100$ initially and double the value until a problem occurs.
   - Solve the equation via LU factorization and QR factorization, respectively.
   - Use IMAGESC to view the matrices and factor matrices.
   - Use MATLAB visualization tool to present the solution.
   - Give comments on the sparsity of the factor matrices, the time for the solution, and the discrepancy between the numerical solutions.

2. Solve the above problems with the following iterative methods

   (a) Jacobi method

   (b) Gauss-Seidel method

   (c) SOR method

3. Give a numerical solution for the decision problem of graph isomorphism. Let $G(V,E)$ be a graph.

(a) Adjacency matrix (note-to-node)

$$A(i,j) = 1, \quad (v_i, v_j) \in E.$$

(b) Incident matrix (edge-node)

$$B(i,j) = -1, \quad B(i,k) = 1, \quad e_i = (v_j, v_k) \in E.$$

(c) Degree matrix

$$D = \text{diag}(\text{degree}(v_i)).$$

(d) Find a relationship between A, B, and D.

(e) Describe the GI in terms of A or B or a composed matrix. Give a numerical solution.