IMAGE RESTORATION

The following matrix equation

$$G = C_x H C_y, \quad G, H \in R^{m \times n},$$

is the discretized model for the observed blurred image G, the blurring operators C_x and C_y, and the image H to be restored.

In an ideal case, the blurring operators are translation invariant. In other words, C_x and C_y are Toeplitz matrices. If the boundary conditions are periodic, C_x and C_y are circulant.

a. Noiseless restoration with nonsingular blurring operators

- Assume that the blurring operators are numerically nonsingular. Describe solution methods for H in the following cases
  - the structures of the operators are not exploited,
  - the operators are Toeplitz,
  - the operators are circulant.

  Give the complexity analysis in memory requirement and in arithmetic operations for each of the methods.

  Experiment with given circulant operators and an image in `ResImage1.mat`.

b. Noiseless restoration with numerically singular blurring operators.

Assume that the blurring operators are numerically singular.

The problem is posed as a LS problem instead

$$\min_H \|\tilde{G} - C_x H C_y\|_F$$

where the matrix norm $\|\cdot\|_F$ is defined as follows

$$\|A\|_F^2 = \sum_j \|Ae_j\|^2.$$

- Describe the LS solution in the above case,
- Provide complexity analysis,

- Experiment with given circulant operators and an image in `ResImage2.mat`.

c. Noisy restoration with nearly singular operators.

In this case, there are errors in the observed image from different sources,

$$\tilde{G} = G + E.$$

The blurring operators have small eigenvalues.

Consider the simplest case that the operators are circulant. Carry out numerical experiments, using the given operators and an image in `ResImage3.mat`,

- ◇ without regulating the weights on the operator eigenvalues.
- ◇ with regulation on the eigenvalues.

Give some explanations to what you have observed.

d. Data files: `ResImage1.mat, ResImage2.mat, ResImage3.mat`