CompSci 102 Discrete Mathematics for CS
Spring 2006 Forbes HW 1

Due Monday, January 30 in class

Questions may continue on the back. Please write clearly. Typed homework is preferable. A good compromise is to type the words and write the math by hand.

I have adhered to the Duke Community Standard in completing this assignment.

Signature: ____

1. (2 pts.) Basics
Complete survey labeled *Beginning of Semester Survey* on Blackboard.

2. (13 pts.) Book problems - Warmup for Recitation

(a) §1.2, Exercise 36
(b) §1.3, Exercise 26
(c) §1.4 Exercise 34
(d) §1.4, Exercise 50

3. (20 pts.) Knights and Knaves
Monique is either a knight or a knave. Knights always tell the truth, and only the truth; knaves always tell falsehoods, and only falsehoods. Someone asks Monique, "Are you a knight?" She replies, "If I am a knight, then I'll stand on my head."

(a) Must Monique stand on her head?
(b) Let's set this up as a problem in propositional logic. Introduce the following propositions:

$$P = \text{"Monique is a knight."}$$
$$Q = \text{"Monique will stand on her head."}$$

Translate what you are given into propositional logic, i.e. rewrite the premises in terms of these propositions.

(c) Using proof by enumeration, prove that your answer from part (a) follow from the premises you wrote in part (b). (No inference rules allowed)

4. (5 pts.) Unicorns!

Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

5. (15 pts.) Inference rules

For each of the following, define proposition symbols for each simple proposition in the argument (for example, $P =$ "Kangaroos live in Australia"). Then write out the logical form of the argument. If the argument form corresponds to a known inference rule, say which it is. If not, show that the proof is correct using truth tables.

(a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

(b) It is hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees today. Therefore, the pollution is dangerous.

(c) Linda is an excellent swimmer. If Linda is an excellent swimmer, than Linda can work as a lifeguard. Therefore, Linda can work as a lifeguard.

(d) Steve will work at a computer company this summer. Therefore, Steve will work at a computer company this summer or he will be a beach bum.

(e) If I work all night on this homework, I will answer all the exercises, If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, I will understand the material.

6. (20 pts.) Setting up a minesweeper problem

Consider the following minesweeper problem:

	1	2	3	4	5
1					
2	1	1	1	1	1

(a) Write out the CNF expressions corresponding to the local constraints $C_{1,2}$, $C_{2,2}$, $C_{3,2}$, $C_{4,2}$, $C_{5,2}$,arising from squares $(1,2)$, $(2,2)$, $(3,2)$, $(4,2)$, $(5,2)$.

(b) Construct a truth table for the problem. Add columns for the expressions $C_{1,2}$, $C_{2,2}$, $C_{3,2}$, $C_{4,2}$, $C_{5,2}$.

(c) Mark those rows that are models of this set of constraints.

(d) Deduce what you can about the unknown squares.

7. (25 pts.) Truth Tables

In class and in the book you saw several truth tables that take two propositions as their input. The operators associated to these tables are called *binary*, because they take two propositions as operands. For instance,

p	q	
T	T	T
T	F	F
F	T	F
F	F	F

is the truth table for the conjunction ("and") of the two propositions p and q, that is, the truth table of $p \wedge q$, so conjunction is a binary operator. How many distinct binary truth tables could one conceivably make up? The answer is sixteen: Truth tables for binary operators all have four rows, and the tables differ from one another by the content of the four elements in the last column. There are sixteen possible ways to fill the last column, and here they are:

p	q	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	r
T	T	T	T	T	T	T	T	T	T	F	F	F	F	F	F	F	F
T	F	T	T	T	T	F	F	F	F	T	T	T	T	F	F	F	F
F	T	T	T	F	F	T	T	F	F	T	T	F	F	T	T	F	F
F	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F

Some of these are familiar: `a` is a tautology, `r` is a contradiction, `b` is $p \vee q$, `c` is $q \rightarrow p$, `e` is $p \rightarrow q$, and so forth. Some others we have never seen before.

Table `i` is called the *nand* of p and q. Let us use a special symbol for it, $p \,|\, q$ (called *Sheffer's stroke*). Let us repeat its truth table here:

p	q	$p \,\vert\, q$
T	T	F
T	F	T
F	T	T
F	F	T

.

From this table, we can see immediately the following logical equivalence:

$$p \,|\, q \equiv \neg(p \wedge q) ,$$

so we can obtain the table for "and" (column `h`) from that of "nand" by inverting the equivalence:

$$p \wedge q \equiv \neg(p \,|\, q) .$$

Write sixteen lines in the following format:

`x` : *expression*

where `x` represents one of the sixteen letters in the column headings of the table above, and *expression* is an implementation of the corresponding truth table using nothing other than the symbols p, q, "nand" operators and parentheses. For example,

`i` : $p \,|\, q$.

There may be different answers for each column. Just give any one. Show your reasoning, and also give the final table neatly written with the columns in alphabetical order. [Hint: Column `m` is the truth table for $\neg p$, which can be implemented as follows:

`m` : $p \,|\, p$

(check with a two-row truth table that this is the case). It is best to build your answer from the easiest cases, and derive harder answers from easier ones.]