# CompSci 102 Discrete Mathematics for CS
# Spring 2006 Forbes HW 6

**1. (15 pts.) A different birthday problem**

(a) In a company of $n$ people, what is the probability that exactly $k$ of them have a birthday on Christmas Day? [Your answer should contain a binomial coefficient, and should be given as a function of $n$ and $k$. Assume that birthdays are independently and uniformly distributed, and ignore the detail of leap years.]

(b) Now suppose $n = 500$. Compute the above probabilities (accurate to four decimal places) for $k = 0, 1, 2, 3, 4, 5, 6$. What is the expectation?

(c) The Poisson approximation for a binomial random variable with parameters $(n, p)$ (meaning that in $n$ independent trails, each results in a success with probability $p$ and failure with probability $1 - p$) is:

$$\Pr(X = i) \approx e^{-\lambda} \frac{\lambda^i}{i!}$$

where $X$ is the number of successes that occur after $n$ trials and $\lambda = np$.

Use the Poisson approximation to estimate the probabilities in part (b), again to four decimal places. How good is the approximation?

**2. (10 pts.) Sequences**

The sequence $s_n = n^2$ for $n = 0, 1, \ldots$ can be defined recursively, that is, by giving its first value and a way to compute $s_{n+1}$ from $s_n$ for $n = 0, 1, \ldots$.

(a) Define $s_n$ recursively. [Hint: there is more than one way to do this.]

(b) Let

$$S_n = \sum_{i=0}^{n} s_i$$

where $s_i$ is the same sequence as before, that is, $s_i = i^2$. Define $S_n$ recursively. [Hint: this is trivial.]

**3. (15 pts.) Hamilton**

Prove by induction that every hypercube has a Hamilton circuit.

**4. (60 pts.) Book Problems**

(a) §6.1, Exercise 20

(b) §6.1, Exercises 54 and 56a

(c) §6.2, Exercise 6

(d) §6.2, Exercise 24

(e) §6.6, Exercise 6

(f) §8.2, Exercise 28

(g) §8.3, Exercise 62

(h) §8.7, Exercise 26

**5. (20 pts.) Extra Credit**

Computations and Explorations 13 on p. 629