Third Homework Assignment

Write the solution to each problem on a single page. The deadline for handing in solutions is February 23.

Question 1. ($20 = 10 + 10$ points). (Problem 3.1-6 in our textbook). Show that $p \oplus q$ is equivalent to $(p \wedge \neg q) \vee (\neg p \wedge q)$. State the corresponding relation in terms of sets and set operations.

Question 2. ($20 = 10 + 10$ points). (Problem 3.2-14 in our textbook). Let x, y, z be variables and p, q logical statements that depend on one variable.

(a) Are the following two compound logical statements equivalent?

1. $(\exists x \in \mathbb{R}\ [p(x)]) \wedge (\exists y \in \mathbb{R}\ [q(y)])$;
2. $\exists z \in \mathbb{R}\ [p(z) \wedge q(z)]$.

(Justify your answer.)

(b) Are the following two compound logical statements equivalent?

1. $(\exists x \in \mathbb{R}\ [p(x)]) \vee (\exists y \in \mathbb{R}\ [q(y)])$;
2. $\exists z \in \mathbb{R}\ [p(z) \vee q(z)]$.

(Justify your answer.)

Question 3. (20 points). (Problem 3.3-6 in our textbook). Is the statement $p \Rightarrow q$ equivalent to the statement $\neg p \Rightarrow \neg q$? (If yes, why? If no, why not?)

Question 4. (20 points). (Problem 3.3-14 in our textbook). Prove that there is no largest prime number. In other words, for every prime number there is another, larger prime number.