Sixth Homework Assignment

Write the solution to each problem on a single page. The deadline for handing in solutions is 22 April 2009.

Question 1. $(20 = 5 + 5 + 5 + 5$ points). Choose ten of your friends, and make a graph where the edges represent two friends being Facebook friends. (Do not include yourself in the graph). Order your friends alphabetically, and label the vertices $v_1, v_2, \ldots, v_{10}$ respectively. This will be most interesting if all of your friends know each other. Now, answer the following questions about the graph that you drew.

  (a) What is the size of the largest clique?
  (b) Find the shortest and longest paths from v_1 to v_{10}.
  (c) Which vertex has the highest degree?
  (d) Use Prim's algorithm to find the minimum spanning tree, and draw that tree.

Question 2. (20 points). (Problem 6.1-14 in our textbook). Are there graphs with n vertices and $n - 1$ edges and no cycles that are not trees? Give a proof to justify your answer.

Question 3. (20 points). Call a simple graph with $n \geq 3$ vertices an *Ore graph* if every pair of non-adjacent vertices has a combined degree of at least n. Is it true that every Ore graph is Hamiltonian? Justify your answer.

Question 4. $(20 = 10 + 10$ points). (Problems 6.4-12 and 13 in our textbook). Prove or give a counterexample:

  (a) Every tree is a bipartite graph.
  (b) A bipartite graph has no odd cycles.

Question 5. $(20 = 5 + 15$ points). Suppose you have n pennies which you arrange flat on a table, without overlap.

  (a) How would you arrange the pennies to maximize the number of pennies that touch each other?
  (b) Prove that the number of touching pairs cannot exceed $3n$.