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Similarity

CPS 296.3: Information Management and Mining

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† Thanks to contents borrowed from Ullman (http://infolab.stanford.edu/~ullman/mining/mining.html)

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Announcements

- Office hours: Mondays and Tuesdays 4-5pm
- Seminar-style (reading/discussion) portion of the course will start in three weeks

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Example: face recognition

- Given a large database of face images (say 1 million), find the most similar faces in the database
- Each image represented by a large number (say 1000) numerical features
  - E.g., some (relatively invariant) value like ratio of nose width to eye width
- Two images are similar if at least some fraction (say ¾) of the features are close
- Many-one problem: given a new face, see if it is close to any of the 1 million old faces
- Many-many problem: find which pairs of the 1 million faces are similar

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A simple solution

- Represent each face by a 1000-vector
- Score similarity by comparing pairs of vectors
- Sort-of okay for many-one problem
- Out of the question for many-many problem
  - $\approx 10^6 \times 10^6 \times 1000 / 2$ numerical comparisons!

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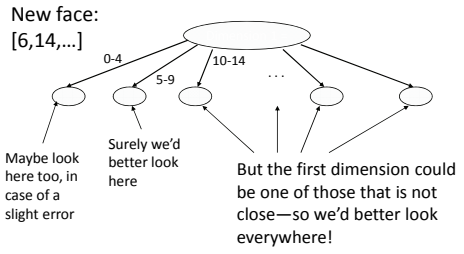
A multidimensional index?



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Example 2: entity resolution

- Given two large sets (say 1 million each) of name-address-phone records, find pairs (one from each set) representing the same person
- Errors of many kinds
  - Typos, missing middle initial, area-code changes, St./Street, Bob/Robert, etc.
- Choose a similarity function between two records
  - E.g., for names, deduct so much for edit distance > 0, so much for missing middle initial, etc.
  - Similarly score addresses and phone numbers
  - Sufficiently high total score → records represent the same entity

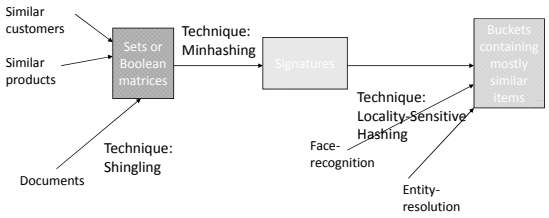
Example 3: mining purchases

- Common pattern: look for sets with relatively large intersection
- E.g., represent a Netflix customer by the set of movies she rented
  - Similar customers have relatively large fraction of their choices in common
- E.g., represent a product by the set of customers who purchased it
  - Similar movies have many renters in common
  - ➲Tricky: Sony & Samsung TV's are "similar," but not typically bought by the same customers

Example 4: similar documents

- Given a body of docs, e.g., the Web, find pairs of docs that have a lot of text in common, e.g.:
  - Mirrors, or approximate mirrors
  - Plagiarism, including large quotations
  - Repetitions of news articles
- Challenge: how to represent a doc so it is easy to compare with others
  - Special cases are easy (how?), e.g., identical docs, or doc contained verbatim in another
  - General case is hard, when many small pieces of one doc appear out of order in another

Roadmap



Representing docs

- Represent a doc by its set of shingles (k-grams)
- Summarize the shingle set by a signature—a small piece of data with the property that similar docs are very likely to have "similar" signatures
- Then, the doc similarity problem becomes the problem of finding similar sets

Shingles

- A k-shingle (k-gram) for a doc is a sequence of k characters that appears in the doc
  - Example: k = 2; doc = abcab
    - Set of 2-shingles = {ab, bc, ca}
    - Another option is to regard shingles as a "bag," and count ab twice
- Long shingles are typically compressed—hashed to (say) 4 bytes
  - Doc = set of hash values of its k-shingles
    - ➲Two docs could (rarely) appear to have shingles in common, when in fact the hash values collided

Next: Minhashing

- Many similarity problems can be couched as finding subsets of some universal set that have large intersection, e.g.:
  - Docs as sets of shingles (or hashes of these shingles)
  - Similar customers/products
- Minhashing: a way of compressing sets into signatures such that signature similarity approximates set similarity

From sets to a Boolean matrix

- Rows = elements of the universal set
- Columns = sets (subsets of the universal set)
- 1 in the row for element e and the column for set S iff $e \in S$

S = {a,b,c,e,f} T = {a,d,f,g,h} U = {b,e}
V = {a,b,c,f,g,h} W = {d,e,f,g}

	S	T	U	V	W
a	1	1	0	1	0
b	1	0	1	1	0
c	1	0	0	1	0
d	0	1	0	0	1
e	1	0	1	0	1
f	1	1	0	1	1
g	0	1	0	1	1
h	0	1	0	1	0

Set similarity

- Jaccard similarity between two sets: ratio of the sizes of their intersection and union
  - Sim(S_1, S_2) = $|S_1 \cap S_2| / |S_1 \cup S_2|$
- Recall: a column = set of rows in which it has 1
- Example:

C_1	C_2
0	1
1	0
1	1
0	0
1	1
0	1

Sim(C_1, C_2) = 2/5 = 0.4

Approach outline

- Compute signatures of columns
  - Read from disk to main memory
- Find similar signatures in main memory
  - Critical: similarity of signatures ≈ similarity of columns
- Optional: check that columns with similar signatures are really similar
  - To remove false positives

Warnings

- Comparing all pairs of signatures may take too much time, even if not too much space
  - A job for Locality-Sensitive Hashing; more later
- These methods can give false negatives
  - And even false positives, if the optional check on the last slide is skipped

Signatures

- Key idea: "hash" column C to a small Sig(C), such that
  - Sig(C) is small enough that all column signatures can fit in main memory
  - Sim(C_1, C_2) is the same as the "similarity" of Sig(C_1) and Sig(C_2)
- Does the following approach work?
  - Pick 100 rows at random, and let Sig(C) be the 100 bits of C in these rows

➲No! Sparse matrix ⇒ most signatures will have all 0's, even though the columns have different 1's elsewhere

Minhashing

- Imagine the rows are permuted randomly
- Define "(min)hash" function $h(C)$ = the index of the first (in the permuted order) row in which column C has 1

➲Use several (100?) independent hash functions to create a signature (of 100 integers)

Minhashing example

Input matrix

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

1	4	3				
3	2	4				
7	1	7				
6	3	6				
2	6	1				
5	7	2				
4	5	5				

Signature matrix

2	1	2	1
2	1	4	1
1	2	1	2

Surprising property

- Given columns C_1 and C_2, rows can be classified into four types:

C_1	C_2
1	1
1	0
0	1
0	0

  - Let n = # of rows, and n_{xx} = # of rows of type xx
  - Sim(C_1, C_2) = n_{11} / (n_{11} + n_{10} + n_{01})
- The probability (over all permutations of rows) that $h(C_1) = h(C_2)$ is the same as Sim(C_1, C_2)

Why?

- # of permutations for which $h(C_1) = h(C_2) = i$:
  - $P(n_{00}, i-1)\ n_{11}\ (n-i)!$
- # of permutations for which the first 1 in either C_1 or C_2 (or both) shows up at index i:
  - $P(n_{00}, i-1)\ (n_{11} + n_{10} + n_{01})\ (n-i)!$
- The ratio is exactly n_{11} / (n_{11} + n_{10} + n_{01})
- A simpler proof?
  - Consider Pr[$h(C_1) = h(C_2)$ | first 1 shows up at index i]

Similarity of signatures

- The similarity between two signatures is the fraction of the rows in which they agree
  - Recall each signature row k corresponds to a hash function h_k
  - Sim(Sig(C_1), Sig(C_2)) = $\sum_k \mathbf{1}(h_k(C_1) = h_k(C_2))$ / K

Signature matrix

2	1	2	1
2	1	4	1
1	2	1	2

Similarities

Implementation challenges

- Suppose the size of the universal set is 1 billion
- Difficult to pick a random permutation of 1..billion
- Representing a random permutation requires 1 billion entries
- Access rows in permuted order leads to thrashing

Ideas

- Instead of truly random permutations, settle for something close to a min-wise independent family of permutations
  - [Broder et al. STOC '98]
  - I.e., if we randomly pick π from this family, then for any C, all its elements have an equal chance to become the first one after applying π
  - E.g., pick (say) 100 hash functions of the form $\pi(x) = ax + b$ mod p (p is the universe size and prime)
- Instead of making the outer loop go over every hash function, scan the Boolean matrix once and process all hash functions

Implementation

keep signature matrix M in memory

for each row r corresponding to an element

  for each column c corresponding to a set

    if c has 1 in row r

      for each permutation π_k

        if $\pi_k(r) < M(k, c)$ then

          $M(k, c) \leftarrow \pi_k(r)$

Example

Row	C_1	C_2
1	1	0
2	0	1
3	1	1
4	1	0
5	0	1

$\pi_1(x) = x \bmod 5$

$\pi_2(x) = 2x+1 \bmod 5$

	Sig1	Sig2
$\pi_1(1) = 1$	1	-
$\pi_2(1) = 3$	3	-
$\pi_1(2) = 2$	1	2
$\pi_2(2) = 0$	3	0
$\pi_1(3) = 3$	1	2
$\pi_2(3) = 2$	2	0
$\pi_1(4) = 4$	1	2
$\pi_2(4) = 4$	2	0
$\pi_1(5) = 0$	1	0
$\pi_2(5) = 1$	2	0

Some more details

- If the input (Boolean matrix) is stored row-by-row, then only one pass is needed
- If the input is stored column-by-column
  - E.g., doc by doc
- Option 1: represent it by (row, column) pairs and sort it once by row
  - External-memory merge sort is pretty scalable
- Option 2?

Finding similar pairs

- Suppose we have in main memory data representing a large number of objects, e.g.:
  - Feature vectors for face images
  - Minhash signatures for docs
- We want to compare each to each, finding those pairs that are sufficiently similar

"Sufficiently similar"

- For finding similar sets
  - Pick a threshold s (< 1) for Jaccard similarity
  - A pair of columns C_1 and C_2 is a candidate pair if their minhash signatures agree in at least fraction s of the rows
- For images, a pair of vectors is a candidate if they differ by at most a small amount t in at least s fraction of the components
- For entity records, a pair is a candidate if the sum of similarity scores of corresponding components exceeds a threshold

Checking all pairs is hard

- E.g., 10^6 columns $\Rightarrow 5 \times 10^{11}$ comparisons, or 6 days at 1 μs /comparison

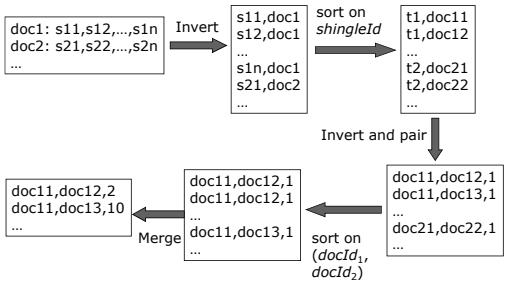
Solutions

- Divide-Compute-Merge (DCM) uses external-memory sorting, merging
- Locality-Sensitive Hashing (LSH) can be carried out in main memory, but admits false negatives

Divide-compute-merge

- Designed for problems where data is presented by column (e.g., shingles and docs)
- At each stage, divide data into batches that fit in main memory
- Operate on individual batches and write out partial results to disk
- Merge partial results from disk

DCM steps



DCM notes

- Streamline adjacent steps
  - E.g., the first invert step can be combined with the first pass of sorting on *shingleId*
- In multi-pass sort-merge, eagerly merge/aggregate
- Eliminate very common shingles
- Eliminate exact-duplicate docs first

Locality-sensitive hashing

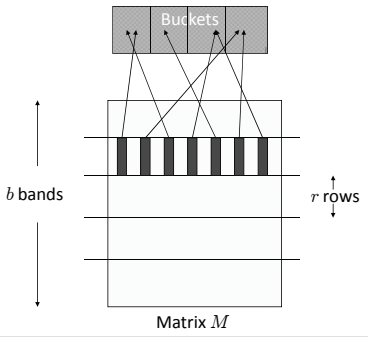
- Hash columns of signature matrix M several times
- Arrange that (only) similar columns are likely to hash to the same bucket
- Candidate pairs are those that hash at least once to the same bucket

Partitioning into bands

- Divide matrix M into b bands of r rows each
- For each band, hash its portion of each column to a hash table
- Candidate column pairs are those that hash to the same bucket for at least one band

➲Tune b and r to catch similar pairs, but few dissimilar pairs

Hashing by band illustrated



A simplifying assumption

- There are enough number of buckets that columns are unlikely to hash to the same bucket unless they are identical in particular band
- Hereafter, we assume "same bucket" = "identical for the band"

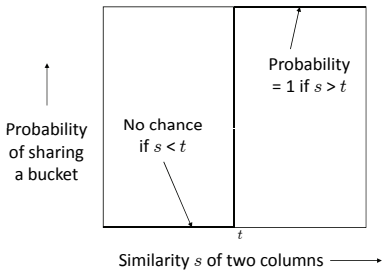
Example

- 100,000 columns, 100 integer signature rows
  - M takes 40MB
  - But 5,000,000,000 pairs of signatures take a while to compare
- Choose 20 bands with 5 integers each
- Suppose C_1, C_2 are 80% similar
  - Pr[C_1, C_2 identical in a given band] = $(0.8)^5 = 0.328$
  - Pr[C_1, C_2 not identical in any band] = $(1 - 0.328)^{20} = 0.00035$
  - I.e., we miss ≈1/3000th of the 80%-similar column pairs
- Suppose C_1, C_2 are only 40% similar
  - Pr[C_1, C_2 identical in a given band] = $(0.4)^5 = 0.01$
  - Pr[C_1, C_2 identical in at least one band] = $1 - (1 - 0.01)^{20} = 0.18$

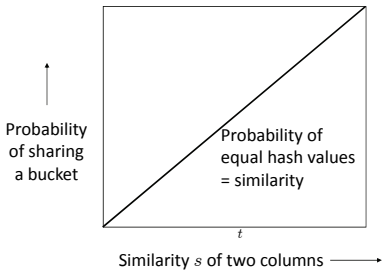
LSH involves a tradeoff

- Pick # of bands and # of rows per band (product = # of minhashes) to balance false positives/negatives
- Example: if we had fewer than 20 bands, the number of false positives would go down, but the number of false negatives would go up
  - Why?

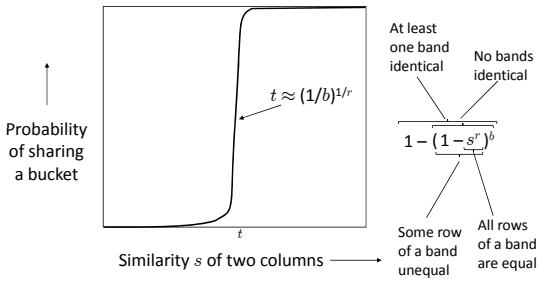
Analysis of LSH: what we want



What testing one row offers



What b bands of r rows offer



LSH summary

- Tune to get almost all pairs with similar signatures, but eliminate most pairs with dissimilar signatures
- In main memory, check that candidate pairs really do have similar signatures
- Optional: in another pass through data, check that the remaining candidate pairs really are similar columns in the input Boolean matrix

LSH for other applications

- Face recognition from 1000 measurements/face
- Entity resolution from name-address-phone records

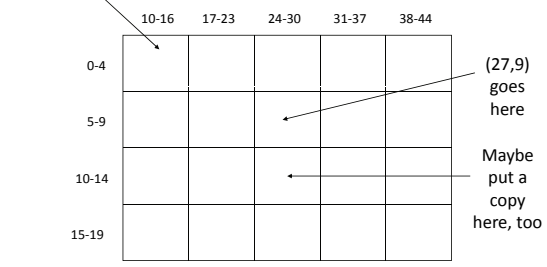
➲General approach: find many hash functions for elements; candidate pairs share a bucket for at least one hash function

Face-rec. "hash" functions

- Pick a set of r of the 1000 features
- Each bucket corresponds to a range of values for each of the r selected features
- Map a vector to the bucket such that all its r selected components are in range
- Optional: if near the edge of a bucket, also map to an adjacent bucket

Example: r = 2

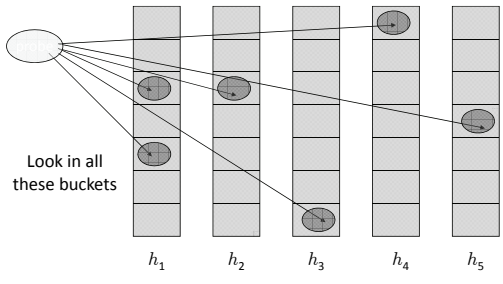
One bucket, for (x, y) if $10 \leq x \leq 16$ and $0 \leq y \leq 4$



Many-one face lookup

- As before, use many different "hash" functions
  - Each based on a different set of the 1000 features
- Each bucket of each function points to images that map to that bucket
- Given an image (the probe), map it to buckets using all functions
- Any member of any one of its buckets is a candidate
- For each candidate, count the # components in which the candidate and probe are close
- Match if # of close components $\geq$ threshold

Hashing the probe



Many-many problem

- Make each pair of images that are in the same bucket (according to any hash function) be a candidate
- Score each candidate pair as for the many-one problem

Matching customer records

Example from Ullman's consulting experience

- A agreed to solicit customers for B, for a fee
- They then had a parting of the ways, and argued over how many customers
- Neither recorded which customers were involved
- B had ~1 million records of all its customers
- A had ~1 million records on customers, some of whom it had signed up for B
- Records had name, address, and phone, but for various reasons, they could be different for the same person

LSH-inspired approach

- Three hash functions: on name, on address, and on phone
- Compare iff records are identical in at least one
- Misses similar records with small differences in all three fields

➲Could also consider fancier LSH, e.g., for string edit distance

Getting more formal with LSH

- Survey: [Andoni & Indyk, CACM 2008]
- A family $\mathcal{H}$ of hash functions mapping $\mathbb{R}^d$ to some universe U is called (R, cR, P_1, P_2)-sensitive if for any two points $p, q \in \mathbb{R}^d$:
  - If $\|p-q\| \leq R$ then $\Pr_{\mathcal{H}}[h(q)=h(p)] \geq P_1$
  - If $\|p-q\| \geq cR$ then $\Pr_{\mathcal{H}}[h(q)=h(p)] \leq P_2$
- ➲Obviously you want $P_1 > P_2$
- Ongoing quest of finding LSH for various metrics
  - Hamming, L_1 (Manhattan), L_2 (Euclidean), L_p, Jaccard ($\rightarrow$ Minhash), EMD, …