CPS 170: Introduction to AI	**Instructor:** Ron Parr
Homework 5	**Due:** Thursday April 15, 2010

1 Decision theory (10 points)

Do problem 16.17 (16.11 in the second edition). You may skip part (a).

2 Decision theory (10 points)

Recall the definition of *value of information.*

(a) Prove that the value of information is nonnegative and order independent.

(b) Explain why it is that some people would prefer not to get some information - for example, not wanting to know the sex of their baby when an ultrasound is done.

(c) A function f on sets is *submodular* if, for any element x and any sets A and B such that $A \subseteq B$, adding x to A gives a greater increase in f than adding x to B:

$$A \subseteq B \Rightarrow (f(A \cup \{x\}) - f(A)) \geq (f(B \cup \{x\}) - f(B)).$$

Submodularity captures the intuitive notion of *diminishing returns.* Is the value of information, viewed as a function f on sets of possible observations, submodular? Prove this or find a counterexample.

3 MDPs (10 points)

Suppose there are two coins. Coin A has probability 0.25 of heads and coin B has probability 0.1 of heads. You are given a chance to play a game with the following rules: At the start of the game, you pick a coin. The selected coin is then flipped until it yields heads. The game then stops. When coin A comes up heads, you get a payoff of 100. When coin B comes up heads you get a payoff of 500. There can be a significant delay between flips, so we'll assume that there is a discount factor of γ applied per time step to future payoffs.

(a) Formulate this problem as an MDP. Indicate the states, actions, and rewards for this MDP. Hints: In some states for your MDP there will be no action choices. You can also view the end of the game as a state with 0 reward and a deterministic transition to itself, i.e., a state which must have value 0.

(b) State the optimal policy for this MDP and justify your answer mathematically. Note that the policy may depend upon the value of γ.

4 MDPs (10 points)

Do problem 17.10 (17.4 in the second edition).

5 Game theory (10 points)

Do problem 17.17 (17.10 in the second edition).

6 Game theory (10 points)

Teams in the National Hockey League historically received 2 points for winning a game and 0 for losing. If the game is tied, an overtime period is played; if nobody wins in overtime, the game is a tie and each team gets 1 point. But league officials felt that teams were playing too conservatively in overtime (to avoid a loss), and it would be more exciting if overtime produced a winner. So in 1999 the officials experimented in mechanism design: the rules were changed, giving a team that loses in overtime 1 point, not 0. It is still 2 points for a win and 1 for a tie.

(a) Was hockey a zero-sum game before the rule change? After?

(b) Suppose that at a certain time t in a game, the home team has probability p of winning in regulation time, probability $0.78 - p$ of losing, and probability 0.22 of going into overtime, where they have probability q of winning, $.9 - q$ of losing, and .1 of tying. Give equations for the expected value for the home and visiting teams.

(c) Imagine that it were legal and ethical for the two teams to enter into a pact where they agree that they will skate to a tie in regulation time, and then both try in earnest to win in overtime. Under what conditions, in terms of p and q, would it be rational for both teams to agree to this pact?

(d) Longley and Sankaran (2005) report that since the rules change, the percentage of games with a winner in overtime went up 18.2%, as desired, but the percentage of overtime games also went up 3.6%. What does that suggest about possible collusion or conservative play after the rule change?