CPS 170
Search I

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With thanks to Vince Conitzer for some slides and figures

What is Search?

- Search is a basic problem-solving method
- We start in an initial state
- We examine states that are (usually) connected by a sequence of actions to the initial state
- Note: Search is a thought experiment

- We aim to find a solution, which is a sequence of actions that brings us from the initial state to the goal state, minimizing cost

Search vs. Web Search

- When we issue a search query using Google, does Google really go poking around the web for us?

- Not in real time!
- Google spiders the web continually, caches results
- Uses page rank algorithm to find the most "popular" web pages that are consistent with your query

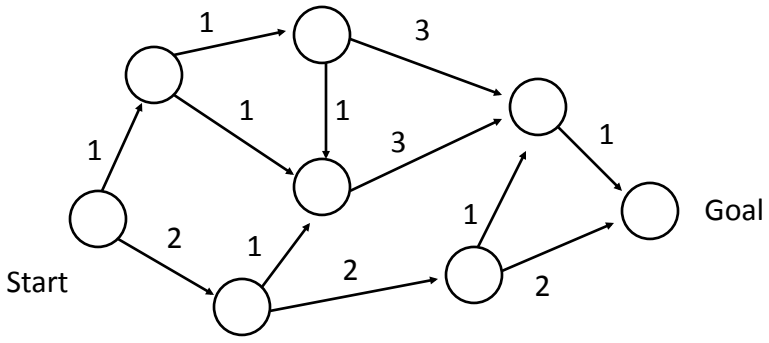
Overview

- Problem Formulation
- Uninformed Search
  - DFS, BFS, IDDFS, etc.
- Informed Search
  - Greedy, A*
- Properties of Heuristics

Problem Formulation

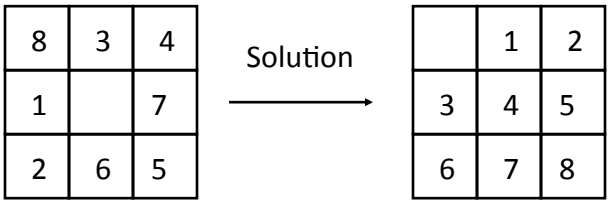
- Four components of a search problem
  - Initial State
  - Actions
  - Goal Test
  - Path Cost
- Optimal solution = lowest path cost to goal

Example: Path Planning



Find shortest route from one city to another using highways.

Example 8(15)-puzzle



Possible Start State

Goal State

Actions: UP, DOWN, RIGHT, LEFT

"Real" Problems

- Robot motion planning
- Drug design
- Logistics
  - Route planning
  - Tour Planning
- Assembly sequencing
- Internet routing

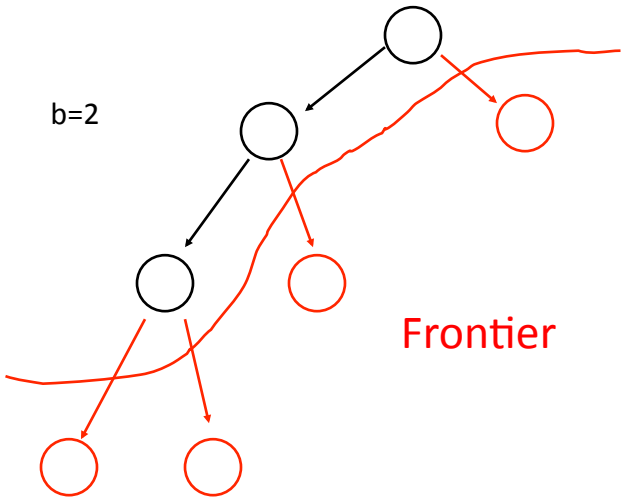
Why Use Search?

- Other algorithms exist for these problems:
  - Dijkstra's Algorithm
  - Dynamic programming
  - All-pairs shortest path
- Use search when it is too expensive to enumerate all states
- 8-puzzle has 362,800 states
- 15-puzzle has 1.3 trillion states
- 24-puzzle has 10^{25} states

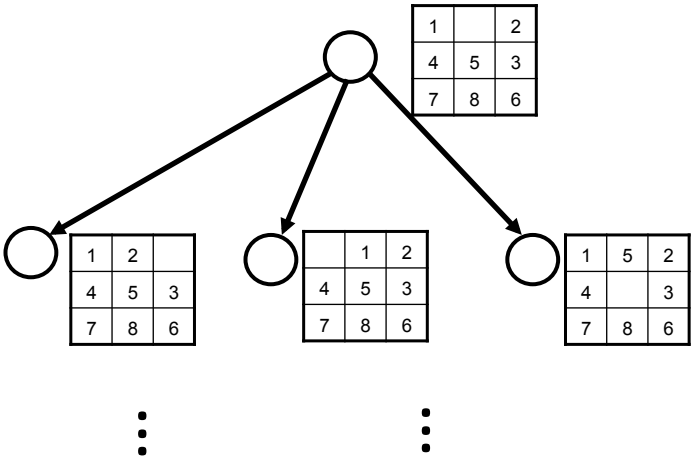
Basic Search Concepts

- Assume a tree-structured space (for now)
- Nodes: Places in search tree
  (states exist in the problem space)
- Search tree: portion of state space visited so far
- Actions: Connect states to next states
- Expansion: Generation of next states for a state
- Frontier: Set of states visited, but not expanded
- Branching factor: Max no. of successors = b
- Goal depth: Depth of *shallowest* goal = d

Example Search Tree



8-puzzle



Generic Search Algorithm

```
Function Tree-Search(problem, Queuing-Fn)

        fringe = Make-Queue(Make-Node(Initial-State(problem)))
        loop do
                if empty(fringe) then return failure
                node = pop(fringe)
                if Goal-Test(problem, state) then return node
                fringe = Add-To-Queue(fringe, expand(node, problem))
        end
```

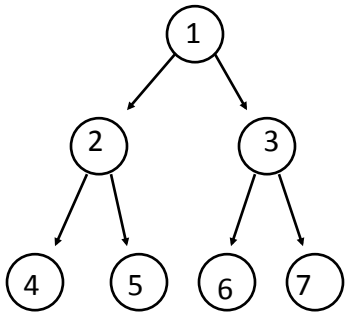
Interesting details are in the implementation of Add-To-Queue

Evaluating Search Algorithms

- Completeness:
  - Is the algorithm guaranteed to find a solution when there is one?
- Optimality:
  - Does the algorithm find the optimal solution?
- Time complexity
- Space complexity

Uninformed Search: BFS

Frontier is a FIFO

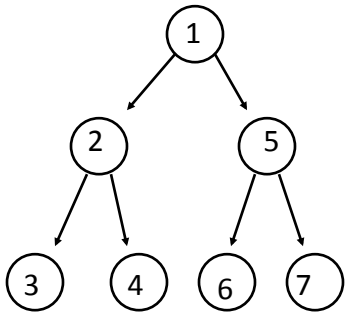


BFS Properties

- Completeness: Y
- Optimality: Y (for uniform cost)
- Time complexity: $O(b^{d+1})$
- Space complexity: $O(b^{d+1})$

Uninformed Search: DFS

Frontier is a LIFO



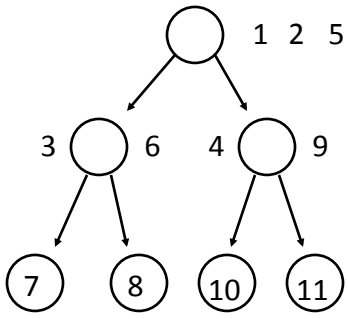
DFS Properties

- Completeness: N (unless tree is finite)
- Optimality: N
- Time complexity: $O(b^{m+1})$ (m = depth we hit, m>d?)
- Space complexity: O(bm)

Iterative Deepening

- Want:
  - DFS memory requirements
  - BFS optimality, completeness
- Idea:
  - Do a depth-limited DFS for depth m
  - Iterate over m

IDDFS



IDDFS Properties

- Completeness: Y
- Optimality: Y (whenever BFS is optimal)
- Time complexity: $O(b^{d+2})$
- Space complexity: O(bd)

IDDFS vs. BFS

Theorem: IDDFS visits no more than twice as many nodes for a binary tree as BFS.

Proof: Assume the tree bottoms out at depth d, BFS visits:

$$2^{d+1} - 1$$

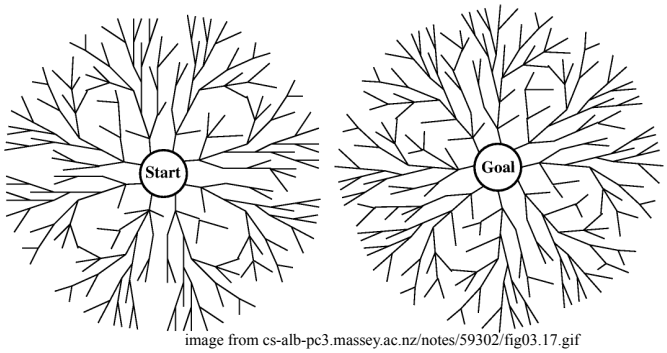
In the worst case, IDDFS does no more than:

$$\sum_{i=1}^{d}(2^{i+1} - 1) = \sum_{i=1}^{d}2^{i+1} - \sum_{i=1}^{d}1 = (2^{d+2} - 2) - d \leq 2(2^{d+1} - 1) = 2 \times BFS(d)$$

What about b-ary trees?

IDDFS relative cost is lower!

Bi-directional Search



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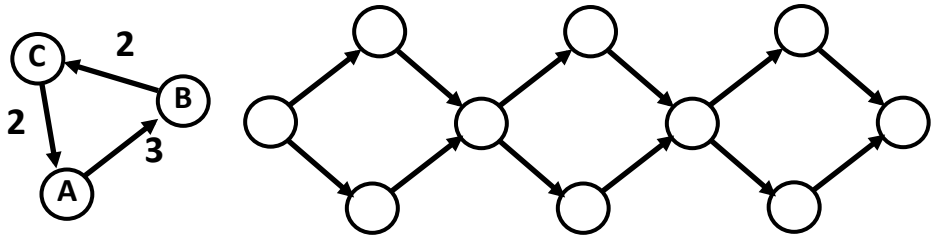
$$b^{d/2} + b^{d/2} << b^{d}$$

Issues with Bi-directional Search

- Uniqueness of goal
  - Suppose goal is parking your car
  - Huge no. of possible goal states (configurations of other vehicles)
- Invertability of actions

What About Repeated States (graphs)



cycles

exponentially large search trees

- Can cause incompleteness or enormous runtimes
- Can maintain list of previously visited states to avoid this
  - If new path to the same state has greater cost, don't pursue it further
  - Leads to time/space tradeoff
- "Algorithms that forget their history are doomed to repeat it" [Russell and Norvig]

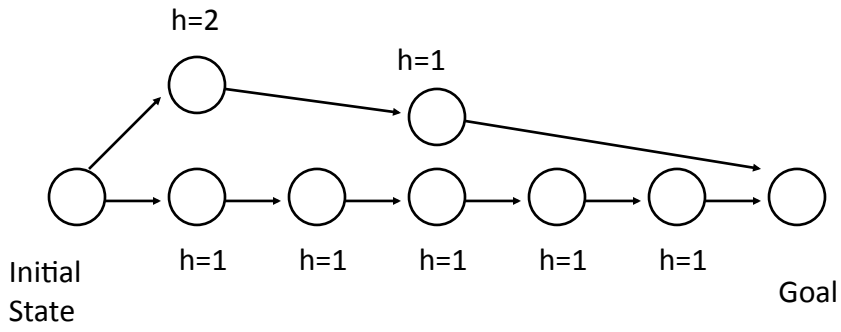
Informed Search

- Idea: Give the search algorithm hints
- Heuristic function: h(x)
- h(x) = estimate of cost to goal from x
- If h(x) is 100% accurate, then we can find the goal in O(bd) time

Greedy Search

- Expand node with lowest h(x)
- Optimal if h(x) is 100% correct
- How can we get into trouble with this?

What Price Greed?



What's broken with greedy search?

A*

- Path cost so far: g(x)
- Total cost estimate: f(x) = g(x) + h(x)
- Maintain frontier as a priority queue
- O(bd) time if h is 100% accurate
- We want h to be an *admissible* heuristic
- Admissible: never overestimates cost

Some A* Properties

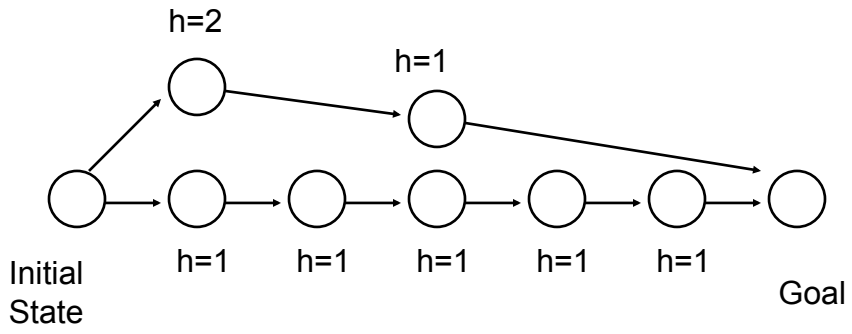
- Implies h(x)=0 if x is a goal state
- Implies f(x)=cost to goal if x is a goal state and x is popped off the queue

- What if h(x)=0 for all x?
  - Is this admissible?
  - What does the algorithm do?

Optimality of A*

- If h is admissible, A* is optimal
- Proof (by contradiction):
  - Suppose a suboptimal solution node n with solution value f(n) > C* is about to be expanded (where C* is optimal)
  - Let n* be a goal state found on optimal path
  - There must be some node n' that is currently in the fringe and on the path to n*
  - We have f(n) > C*, and f(n') = g(n') + h(n') ≤ C*
  - But then, n' should be expanded first (contradiction)

Does A* fix the greedy problem?



A* is optimally efficient

- A* is optimally efficient: Any other optimal algorithm must expand at least the nodes A* expands
- Proof:
  - Besides solution, A* expands the nodes with g(n)+h(n) < C*
    - Assuming it does not expand non-solution nodes with g(n)+h(n) = C*
  - Any other optimal algorithm must expand at least these nodes (since there may be a better solution there)
- Note: This argument assumes that the other algorithm uses the same heuristic h

Properties of Heuristics

- h2 dominates h1 if h2(x)>h1(x) for all x
- Does this mean that h2 is better?

- Suppose you have multiple admissible heuristics. How do you combine them?

Designing heuristics

- One strategy for designing heuristics: relax the problem
- *"Number of misplaced tiles"* heuristic corresponds to relaxed problem where tiles can jump to any location, even if something else is already there
- *"Sum of Manhattan distances"* corresponds to relaxed problem where multiple tiles can occupy the same spot
- The ideal relaxed problem is
  - easy to solve,
  - not much cheaper to solve than original problem
- Some programs can successfully automatically create heuristics