CPS 170
Alternative Search Techniques

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With thanks to Vince Conitzer for LP,(M)IP examples.

Overview

- Memory-bounded Search
- Local Search and Optimization
- Searching with Incomplete Information

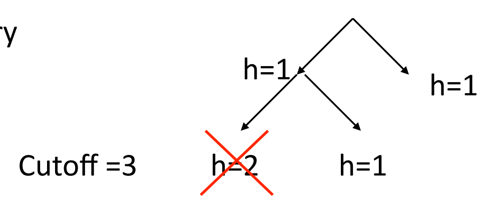
Memory-bounded Search: Why?

- We run out of memory before we run out of time.
- Problem: Need to remember entire search horizon
- Solution: Remember only a partial search horizon

- Issue: Maintaining optimality, completeness
- Issue: How to minimize time penalty

Attempt 1: IDA*

- Iterative deepening A*
- Idea: Like IDDFS, but use the f cost as a cutoff
  - Cutoff all searches with f > 1, then f > 2, f > 3, etc.
  - Motivation: Cut off bad-looking branches early
- Problems:
  - Excessive node regeneration
  - Can still use a lot of memory

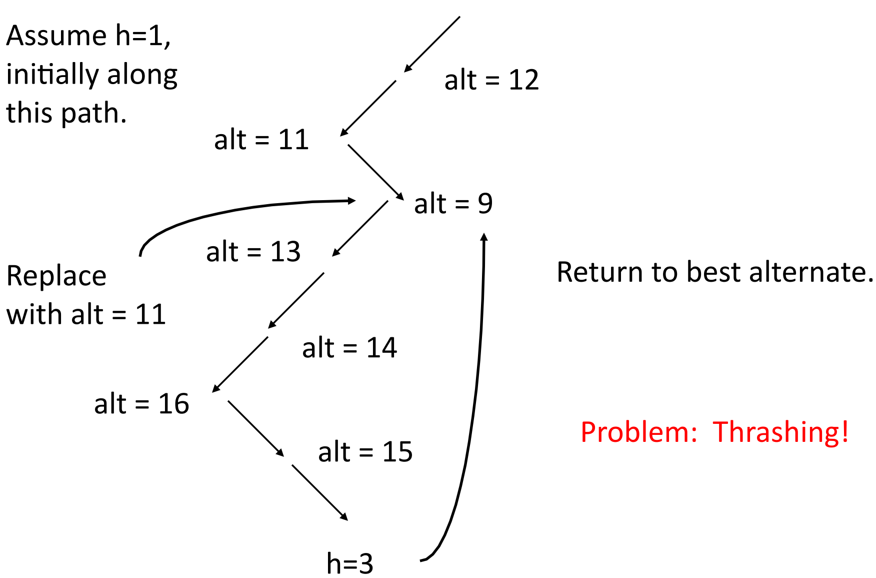


Attempt 2: RBFS

- Recursive best first search
- Objective: Linear space

- Idea: Remember best alternative
- Rewind, try alternatives if “best first” path gets too expensive
- Remember costs on the way back up

RBFS

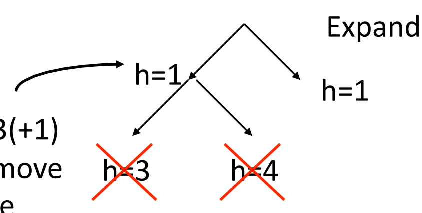


SMA*

- Idea: Use all of available memory
- Discard the *worst* leaf when memory starts to run out, to make room for new leaves
- Values get backed up to parents
- Optimal if solution fits in memory
- Complete
- Thrashing still possible

Painful to implement ☹

Replace
with h=3(+1)
if we remove
this node

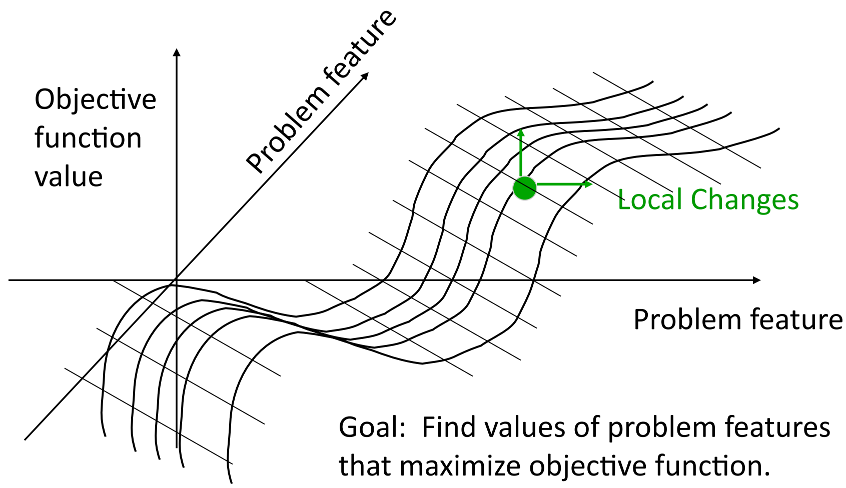


Optimization

- Solution is more important than path
- Interested in minimizing or maximizing some function of the problem state
  - Find a protein with a desirable property
  - Optimize circuit layout
  - Satisfy requirements for your major

- History of search steps not worth the trouble

State Space Landscape



Goal: Find values of problem features that maximize objective function.

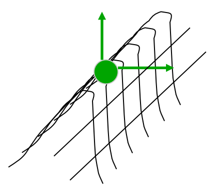
Note: This is conceptual. Often this function is not smooth.

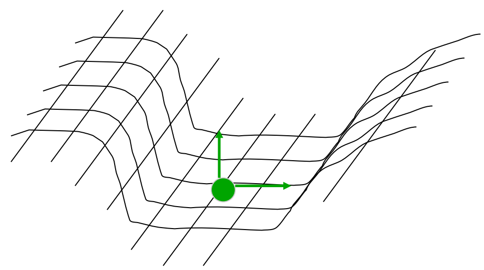
Hill Climbing

- Idea: Try to climb up the state space landscape to find a setting of the problem features with high value.
- Approaches:
  - Steepest ascent
  - Stochastic – pick one of the good ones
  - First choice
- This is a *greedy* procedure

Limitations of Hill Climbing

- Local maxima
- Ridges – direction of ascent is at 45 degree angle to any of the local changes
- Plateaux – flat expanses





Getting Unstuck

- Random restarts
- Simulated annealing
  - Take downhill moves with small probability
  - Probability of moving downhill decreases with
    - Number of iterations
    - Steepness of downhill move
  - If system is "cooled" slowly enough, will find global optimal w.p. 1
  - Motivated by the annealing of metals and glass

Genetic Algorithms

- GAs are hot in some circles
- Biological metaphors to motivate search
- Organism is a word from a finite alphabet (organisms = states)
- Fitness of organism measures its performance on task (fitness = objective)
- Uses multiple organisms (parallel search)
- Uses mutation (random steps)

Crossover

Crossover is a distinguishing feature of GAs:

Randomly select organisms for "reproduction" in accordance with their fitness. More "fit" individuals are more likely to reproduce.

Reproduction is sexual and involves *crossover*:

Organism 1: 1 1 0 0 1 0 0 1 0

Organism 2: 0 0 0 1 0 1 1 1 0

Offspring: 1 1 0 0 1 1 1 1 0

Is this a good idea?

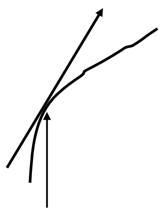
- Has worked well in some examples
- Can be very brittle
  - Representations must be carefully engineered
  - Sensitive to mutation rate
  - Sensitive to details of crossover mechanism
- For the same amount of work, stochastic variants of hill climbing often do better
- Hard to analyze; needs more rigorous study

Continuous Spaces

- In continuous spaces, we don't need to "probe" to find the values of local changes
- If we have a closed-form expression for our objective function, we can use the calculus
- Suppose objective function is: $f(x_1, y_1, x_2, y_2, x_3, y_3)$
- Gradient tells us direction and steepness of change

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

Following the Gradient


x

$$\mathbf{x} = (x_1, y_1, x_2, y_2, x_3, y_3)$$

$$\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x})$$

For sufficiently small step sizes, this will converge to a local optimum.

If gradient is hard to compute:
- Compute empirical gradient
- Compare with classical hill climbing

Constrained Optimization

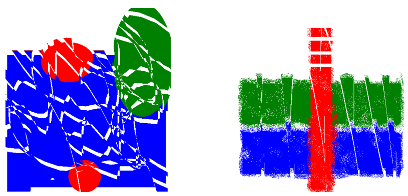
- Don’t forget about the easier cases
  - If the objective function is linear, things are easier
  - If linear constraints, solve as a linear program:
  - Maximize (minimize):

$$f(\mathbf{x})$$

  - Subject to: $\mathbf{Ax} \leq \mathbf{b}$ $\boxed{(\mathbf{Ax} \geq \mathbf{b})}$
  - Can be done in polynomial time
  - Can solve some quadratic programs in poly time

Linear programs: example

- Make reproductions of 2 paintings



- Painting 1:
  - Sells for \$30
  - Requires 4 units of blue, 1 green, 1 red
- Painting 2
  - Sells for \$20
  - Requires 2 blue, 2 green, 1 red
- We have 16 units blue, 8 green, 5 red

maximize $3x + 2y$

subject to

$4x + 2y \leq 16$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$

Solving the linear program graphically

maximize $3x + 2y$

subject to

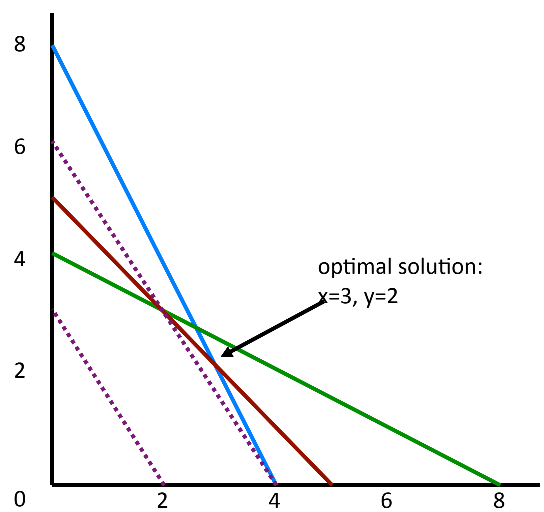
$4x + 2y \leq 16$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$



Modified LP

maximize 3x + 2y

subject to

4x + 2y ≤ 15

x + 2y ≤ 8

x + y ≤ 5

x ≥ 0

y ≥ 0

Optimal solution: x = 2.5, y = 2.5

Solution value = 7.5 + 5 = 12.5

Half paintings?

Integer (linear) program

maximize 3x + 2y

subject to

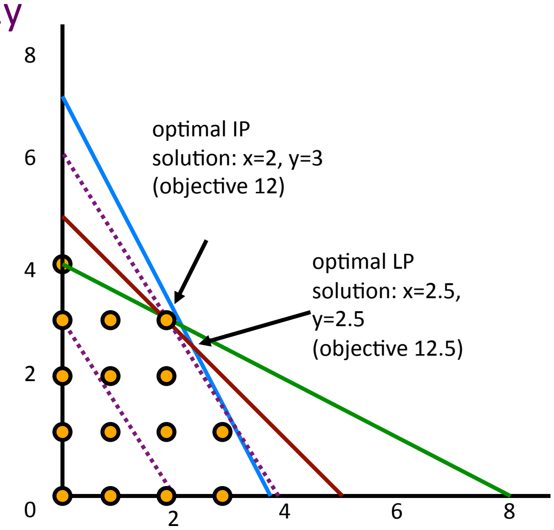
4x + 2y ≤ 15

x + 2y ≤ 8

x + y ≤ 5

x ≥ 0, integer

y ≥ 0, integer



Mixed integer (linear) program

maximize 3x + 2y

subject to

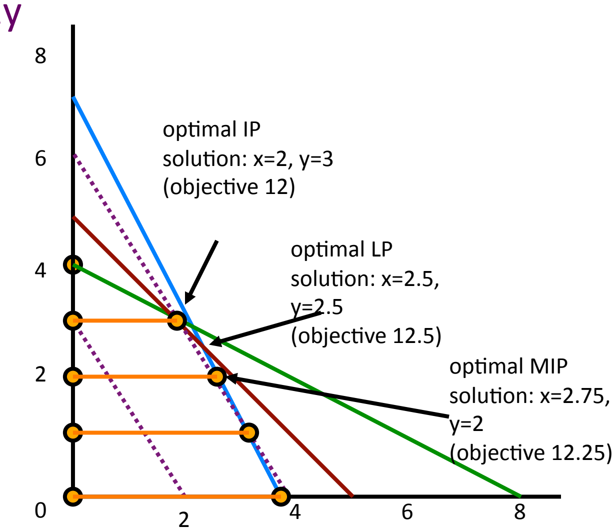
4x + 2y ≤ 15

x + 2y ≤ 8

x + y ≤ 5

x ≥ 0

y ≥ 0, integer



Solving linear/integer programs

- Linear programs can be solved efficiently
  - Simplex, ellipsoid, interior point methods…
- (Mixed) integer programs are NP-hard to solve
  - Quite easy to model many standard NP-complete problems as integer programs (try it!)
  - Search type algorithms such as branch and bound
- Standard packages for solving these
  - GNU Linear Programming Kit, CPLEX, …
- LP relaxation of (M)IP: remove integrality constraints
  - Gives upper bound on MIP (~admissible heuristic)

Searching with Partial Information

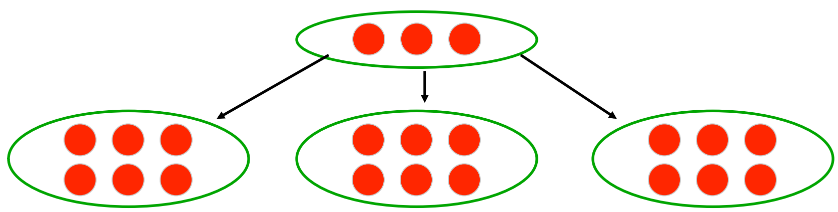
- Multiple state problems
  - Several possible initial states
- Contingency problems
  - Several possible outcomes for each action
- Exploration problems
  - Outcomes of actions not known *a priori*, must be discovered by trying them

Example

- Initial state may not be detectable
  - Suppose sensors for a nuclear reactor fail
  - Need *safe* shutdown sequence despite ignorance of some aspects of state
- This complicates search *enormously*
- In the worst case, contingent solution could cover the entire state space

State Sets

- Idea:
  - Maintain a set of candidate states
  - Each search node represents a set of states
  - Can be hard to manage if state sets get large
- If states have probabilistic outcomes, we maintain a probability distribution over states



Searching in Unknown Environments

- What if we don't know the consequences of actions before we try them?
- Often called on-line search
- Goal: Minimize competitive ratio
  - Actual distance/distance traveled if model known
  - Problematic if actions are irreversible
  - Problematic if links can have unbounded cost

Conclusions and Parting Thoughts

- There are search algorithms for almost every situation
- Many problems can be formulated as search
- While search is a very general method, it can sometimes outperform special-purpose methods