# CPS 296.3:Algorithms in the Real World

**Data Compression III**



## Compression Outline

**Introduction**: Lossy vs. Lossless, Benchmarks, …
**Information Theory**: Entropy, etc.
**Probability Coding**: Huffman + Arithmetic Coding
**Applications of Probability Coding**: PPM + others
**Lempel-Ziv Algorithms**:
- LZ77, gzip,
- LZ78, compress (Not covered in class)

**Other Lossless Algorithms:** Burrows-Wheeler
**Lossy algorithms for images:** JPEG, MPEG, ...
**Compressing graphs and meshes:** BBK



## Lempel-Ziv Algorithms

LZ77 (Sliding Window)

**Variants**: LZSS (Lempel-Ziv-Storer-Szymanski)
**Applications**: `gzip`, Squeeze, LHA, PKZIP, ZOO

LZ78 (Dictionary Based)

**Variants**: LZW (Lempel-Ziv-Welch), LZC
**Applications**: `compress`, GIF, CCITT (modems), ARC, PAK

Traditionally LZ77 was better but slower, but the gzip version is almost as fast as any LZ78.



## LZ77: Sliding Window Lempel-Ziv

Cursor

| a | a | c | a | a | c | a | b | c | a | b | a | b | a | c |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

Dictionary (previously coded) | Lookahead Buffer

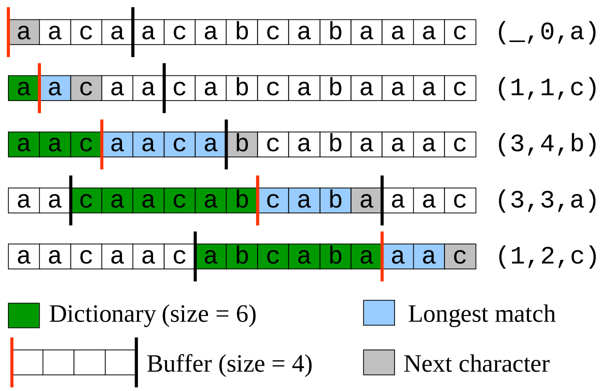
Dictionary and buffer "windows" are fixed length and slide with the cursor

Repeat:

Output (*p*, *l*, **c**) where
*p* = position of the longest match that starts in the dictionary (relative to the cursor)
*l* = length of longest match
**c** = next char in buffer beyond longest match

Advance window by *l* + 1

# LZ77: Example






# LZ77 Decoding

Decoder keeps same dictionary window as encoder.

For each message it looks it up in the dictionary and inserts a copy at the end of the string

What if *l* > p? (only part of the message is in the dictionary.)

E.g. dict = `abcd`, codeword = `(2,9,e)`

- Simply copy from left to right
```
for (i = 0; i < length; i++)
  out[cursor+i] = out[cursor-offset+i]
```
- Out = `abcdcdcdcdcdce`

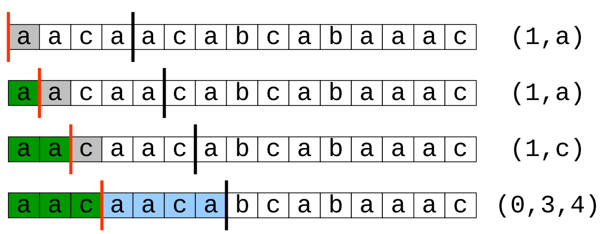


# LZ77 Optimizations used by gzip

LZSS: Output one of the following two formats

**`(0, position, length)`** or **`(1,char)`**

Uses the second format if length < 3.

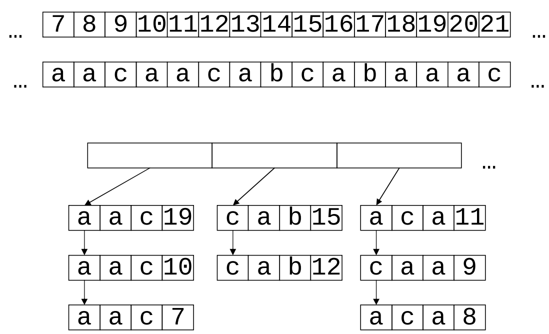





# Optimizations used by gzip (cont.)

1. Huffman code the positions, lengths and chars
2. Non greedy: possibly use shorter match so that next match is better
3. Use a hash table to store the dictionary.
   - Hash keys are all strings of length 3 in the dictionary window.
   - Find the longest match within the correct hash bucket.
   - Puts a limit on the length of the search within a bucket.
   - Within each bucket store in order of position

## The Hash Table






## Theory behind LZ77

The Sliding Window Lempel-Ziv Algorithm is Asymptotically Optimal, A. D. Wyner and J. Ziv, Proceedings of the IEEE, Vol. 82. No. 6, June 1994.

Will compress long enough strings to the source entropy as the window size goes to infinity.

Source entropy for a substring of length n is given by:

$$H_n = \sum_{X \in A^n} p(X) \log \frac{1}{p(X)}$$

Uses logarithmic code (e.g. gamma) for the position.

Problem: "long enough" is really **really** long.



## Comparison to Lempel-Ziv 78

Both LZ77 and LZ78 and their variants keep a "dictionary" of recent strings that have been seen.

The differences are:

- How the dictionary is stored (LZ78 is a trie)
- How it is extended (LZ78 only extends an existing entry by one character)
- How it is indexed (LZ78 indexes the nodes of the trie)
- How elements are removed



## Lempel-Ziv Algorithms Summary

Adapts well to changes in the file (e.g. a Tar file with many file types within it).

Initial algorithms did not use probability coding and performed poorly in terms of compression. More modern versions (e.g. gzip) do use probability coding as "second pass" and compress much better.

The algorithms are becoming outdated, but ideas are used in many of the newer algorithms.

## Compression Outline

**Introduction**: Lossy vs. Lossless, Benchmarks, …
**Information Theory**: Entropy, etc.
**Probability Coding**: Huffman + Arithmetic Coding
**Applications of Probability Coding**: PPM + others
**Lempel-Ziv Algorithms**: LZ77, gzip, compress, …
**Other Lossless Algorithms:**
- Burrows-Wheeler
- ACB

**Lossy algorithms for images:** JPEG, MPEG, …
**Compressing graphs and meshes:** BBK



## Burrows -Wheeler

Currently near best "balanced" algorithm for text

Breaks file into fixed-size blocks and encodes each block separately.

**For each block:**

- Sort each character by its full context. This is called the **block sorting transform**.
- Use **move-to-front transform** to encode the sorted characters.

The ingenious observation is that the decoder only needs the sorted characters and a pointer to the first character of the original sequence.



## Burrows Wheeler: Example

Let's encode: **decode**

Context "wraps" around. Last char is most significant.

| Context | Char |
|---|---|
| ecode | d |
| coded | e |
| odede | c |
| dedec | o |
| edeco | d |
| decod | e |

All rotations of input

**Sort Context** →

| Context | Output |
|---|---|
| dedec | o |
| coded | e |
| decod | e |
| odede | c |
| ecode | d ⇐ |
| edeco | d |



## Burrows Wheeler Decoding

Key Idea: Can construct entire sorted table from sorted column alone! First: sorting the output gives last column of context:

| Context | Output |
|---|---|
| c | o |
| d | e |
| d | e |
| e | c |
| e | d |
| o | d |

## Burrows Wheeler Decoding

Now sort pairs in last column of context and output column to form last two columns of context:

| Context | Output |
|---|---|
| c | o |
| d | e |
| d | e |
| e | c |
| e | d |
| o | d |

→

| Context | Output |
|---|---|
| ec | o |
| ed | e |
| od | e |
| de | c |
| de | d |
| co | d |



## Burrows Wheeler Decoding

Repeat until entire table is complete. Pointer to first character provides unique decoding.

| Context | Output |
|---|---|
| dedec | o |
| coded | e |
| decod | e |
| odede | c |
| ecode | d ⇐ |
| edeco | d |

Message was d in first position, preceded in wrapped fashion by ecode: decode.



## Burrows Wheeler Decoding

Optimization: Don't really have to rebuild the whole context table.

| Context | Output |
|---|---|
| dedec | o |
| code$\boxed{d_1}$ | $\boxed{e_1}$ |
| decod$_2$ | $e_2$ |
| odede$_1$ | c |
| ecode$_2$ | $d_1$ ⇐ |
| edeco | $d_2$ |

What character comes after the first character, $d_1$?

Just have to find $d_1$ in last column of context and see what follows it: $e_1$.

Observation: instances of same character of output appear in same order in last column of context. (Proof is an exercise.)



## Burrows-Wheeler: Decoding

The "rank" is the position of a character if it were sorted using a stable sort.

| Context | Output | | Rank |
|---|---|---|---|
| c | o | | 6 |
| d | e | | 1 |
| d | e | | 4 |
| e | c | | 5 |
| e | d | ⇐ | 2 |
| o | d | | 3 |

## Burrows-Wheeler Decode

Function BW_Decode(In, Start, n)
  *S* = MoveToFrontDecode(In,n)
  *R* = Rank(S)
  *j* = Start
  **for** *i=1* **to** *n* **do**
    Out[*i*] = S[*j*]
    *j* = *R*[*j*]

Rank gives position of each char in sorted order.



## Decode Example

| S | Rank(S) | | Out |
|---|---|---|---|
| $o_4$ | 6 | | $d_1$ ⇐ |
| $e_2$ | 4 | | $e_2$ |
| $e_6$ | 5 | | $c_3$ |
| $c_3$ | 1 | | $o_4$ |
| $d_1$ | 2 | ⇐ | $d_5$ |
| $d_5$ | 3 | | $e_6$ |



## Overview of Text Compression

**PPM** and **Burrows-Wheeler** both encode a single character based on the immediately preceding context.

**LZ77** and **LZ78** encode multiple characters based on matches found in a block of preceding text

Can you **mix these ideas**, *i.e.,* code multiple characters based on immediately preceding context?

- **BZ** does this, but they don't give details on how it works – current best compressor
- **ACB** also does this – close to best



## ACB (Associate Coder of Buyanovsky)

Keep dictionary sorted by context (the last character is the most significant)

- Find longest match for context
- Find longest match for contents
- Code
  - Distance between matches in the sorted order
  - Length of contents match

Has aspects of Burrows-Wheeler, and LZ77

| Context | Contents |
|---|---|
| | decode |
| dec | ode |
| d | ecode |
| decod | e |
| de | code |
| deco | de |