

Thursday, Jan. 26
26 points

1. (4 pts) Let p , q , and r be the following propositions.
 p : You have the flu.
 q : You miss the final exam.
 r : You pass the course.
Express each of the following propositions as English sentences.
 - (a) $q \rightarrow \neg r$
 - (b) $(p \wedge q) \vee (\neg q \vee r)$
2. (4 pts) Determine if these conditional statements are true or false.
 - (a) if $5+1 = 7$ then Durham is the capital of NC.
 - (b) if Durham is not the capital of NC then $5+1 = 7$
3. (3 pts) Show that $(p \rightarrow r) \vee (q \rightarrow r)$ is logically equivalent to $(p \wedge q) \rightarrow r$ with a truth table.
4. (3 pts) There is an island of knights and knaves. The knights always tell the truth. The knaves always lie. You encounter two people, A and B, and they both make a statement. Determine if you can tell what type of people each is or not and reason why.
 A says "I am a knight", B says "I am a knight"
5. (3 pts) Same problem setup as 4) but now there is also a third type of person, a spy who can either lie or tell the truth.
 You encounter three people A, B, and C. You know one is a knight, one is a knave and one is a spy. Each of the three people knows the type of person each of the other two people is. Determine if there is a unique solution of who the knight, knave or spy is. If not list at least two possibilities.
 A says "I am the knight.", B says "A is not the knave", C says "B is not the knave".
6. (3 pts) Determine if the following is satisfiable.
 $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$
7. (3 pts) Show that $(p \rightarrow r) \vee (q \rightarrow r)$ is logically equivalent to $(p \wedge q) \rightarrow r$ by developing a series of logical equivalences.
8. (3 pts) Show that $((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r$ is a tautology by developing a series of logical equivalences.