

Thursday, Feb. 2
20 points

1. (2 pts) Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.
2. (4 pts) Let $C(x)$ be the statement "x has a cat", $D(x)$ be the statement "x has a dog", and let $F(x)$ be the statement "x has a ferret". Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.
 - (a) A student in your class has a cat, a dog and a ferret.
 - (b) Some student in your class has a cat and a ferret, but not a dog.
3. (4 pts) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.
 - (a) $\forall x(x^2 + 2 \geq 1)$
 - (b) $\exists x(x^2 = -1)$
4. (4 pts) Let $F(x,y)$ be the statement "x can fool y", where the domain consists of all people in the world. Use quantifiers to express each of these statements.
 - (a) Everybody can fool Fred.
 - (b) There is no one who can fool everybody.
5. (6 pts) For each of the following, define proposition symbols for each simple proposition in the argument (for example, $P = \text{"Kangaroos live in Australia"}$). Then write out the logical form of the argument. If the argument form corresponds to a known inference rule, say which it is. If not, show that the proof is correct using truth tables.
 - (a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
 - (b) Linda is an excellent swimmer. If Linda is an excellent swimmer, then Linda can work as a lifeguard. Therefore, Linda can work as a lifeguard.
 - (c) If I work all night on this homework, I will answer all the exercises, If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, I will understand the material.