

PART 1 due Thursday, Feb. 9 at midnight
PART 2 due Thursday, Feb. 9 at the beginning of class
33 points

PART 1:

For part 1 use JFLAP to create jflap files. Submit a README file along with the JFLAP files. The README file should have your name, the name of anyone you got help from or consulted with, and any comments you have on JFLAP.

The JFLAP software is available for free at jflap.org.

You can submit using Eclipse or you can submit through a web page (there is a link on the CompSci 102 assignment web page).

1. Let $\Sigma = \{0, 1\}$. For each of the following languages, write a DFA that recognizes it.

- (a) (4 pts) $L_1 = \{w : w \text{ contains the substring } 010\}$

Name this DFA problem1a.jff

- (b) (4 pts) L_2 is the language that consists of all strings such that between every two 1's, there is an even number of 0's.

Name this DFA problem1b.jff

- (c) (5 pts) $\mathcal{L} = \{w \mid w \text{ contains an even number of 1s or exactly one 0}\}$. Examples in \mathcal{L} : 11, 10, 101, 00, ε . Examples not in \mathcal{L} : 111, 100, 1.

Name this DFA problem1c.jff

2. (5 pts) DFA's "can't count," but they can verify addition. Consider an alphabet with rather interesting symbols:

$$\Sigma = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

which represent columns which could appear when writing the addition and result of two binary numbers. An input of length n represents the addition of two n -bit numbers where the input is given from least to most significant bit. The first number's binary representation is in the top row, the second number's representation is in the middle row, and the result is in the bottom row. Let \mathcal{L} be the language of strings over this alphabet which represent correct addition statements as described above. As an example, correctly representing $4 + 2 = 6$ in this scheme would give the following string (first input symbol on the left)

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

The following input would represent $4 + 5 = 8$, and thus this string should not be in \mathcal{L} :

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Write a DFA which accepts \mathcal{L} . Keep your DFA small; machines with more than 5 states will be penalized.

Name this DFA problem2.jff

PART 2:

1. (2 pts) How many different elements does $A \times B \times C$ have if A has m elements, B has n elements, and C has p elements?
2. (3 pts) Explain why $(A \times B) \times (C \times D)$ and $A \times (B \times C) \times D$ are not the same.
3. (3 pts) Prove the first absorption law from Table 1 by showing that if A and B are sets, then $A \cup (A \cap B) = A$.
4. (3 pts) Let A , B , and C be sets. Show that $(A - B) - C = (A - C) - (B - C)$.
5. (4 pts) Blue Eyes

Note: The following problem is copied verbatim from http://xkcd.com/blue_eyes.html.

A group of people with assorted eye colors live on an island. They are all perfect logicians – if a conclusion can be logically deduced, they will do it instantly. No one knows the color of their eyes. Every night at midnight, a ferry stops at the island. Any islanders who have figured out the color of their own eyes then leave the island, and the rest stay. Everyone can see everyone else at all times and keeps a count of the number of people they see with each eye color (excluding themselves), but they cannot otherwise communicate. Everyone on the island knows all the rules in this paragraph.

On this island there are 100 blue-eyed people, 100 brown-eyed people, and the Guru (she happens to have green eyes). So any given blue-eyed person can see 100 people with brown eyes and 99 people with blue eyes (and one with green), but that does not tell him his own eye color; as far as he knows the totals could be 101 brown and 99 blue. Or 100 brown, 99 blue, and he could have red eyes.

The Guru is allowed to speak once (let's say at noon), on one day in all their endless years on the island. Standing before the islanders, she says the following:

"I can see someone who has blue eyes."

Who leaves the island, and on what night?

There are no mirrors or reflecting surfaces, nothing dumb. It is not a trick question, and the answer is logical. It doesn't depend on tricky wording or anyone lying or guessing, and it doesn't involve people doing something silly like creating a sign language or doing genetics. The Guru is not making eye contact with anyone in particular; she's simply saying "I count at least one blue-eyed person on this island who isn't me."

And lastly, the answer is not "no one leaves."