CPS 102 **Homework 4** **Spring 2012**

Due Thursday, Feb. 16 at the beginning of class
33 points

1. (2 pts) Let A and B be sets. Show that $A \cup (B - A) = A \cup B$

2. (1 pt) Draw the Venn Diagram for $\overline{A} \cap \overline{B} \cap \overline{C}$.

3. (1 pt) Suppose that the Universal set is U $= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Express the following set as a bit string where the ith bit in the string is 1 if i is in the set and 0 otherwise.

   $\{1, 3, 6, 10\}$

4. (2 pts) How can the union of n sets that are all subsets of the universal set U be found using bit strings?

5. (2 pts) Determine whether the function f from $\{a, b, c, d\}$ to itself is one-to-one, where $f(a) = b$, $f(b) = b$, $f(c) = d$, $f(d) = c$.

6. (2 pts) Determine whether $f : \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$ is onto if $f(m, n) = |m| - |n|$.

7. (2 pts) Let f be the function from $\mathbf{R}$ to $\mathbf{R}$ defined by $f(x) = x^2$. Find $f^{-1}(\{x|x > 4\})$.

8. (2 pts) For the following sequence, find a recurrence relation satisfied by the sequence

   $a_n = n + (-1)^n$

9. (2 pts) Find the solution to the recurrence relation with the given initial condition. Use an iterative approach.

   $a_n = a_{n-1} - n$, $a_0 = 4$

10. (2 pts) For the following list of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula of rule is correct, determine the next three terms of the sequence.

    7, 11, 15, 19, 23, 27, 31, 35, 39, 43, ...

11. (1 pt) What is the value of the following sum where $S = \{1, 3, 5, 7\}$?

    $\sum_{j \in S} j^2$

12. (2 pts) Describe an algorithm that determines whether a function from a finite set to another finite set is one-to-one.

13. (2 pts) Devise an algorithm that finds all terms of a finite sequence of integers that are greater than the sum of all previous terms of the sequence. Give the worst-case big-O running time of your algorithm.

14. (2 pts) Show that $(x^3 + 2x)/(2x+1)$ is $O(x^2)$.

15. (2 pts) Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions.

    (a) $f(x) = 3x^5 + (\log x)^4$

    (b) $f(x) = (x^3 + 5\log x)/(x^4 + 1)$

16. (2 pts) Show that $x \log x$ is $O(x^2)$, but that x^2 is not $O(x \log x)$.

17. (2 pts) Suppose that $f(x)$ is $O(g(x))$. Does it follow that $2^{f(x))}$ is $O(2^{g(x)})$?

18. (2 pts) Show that $\lceil xy \rceil$ is $\Omega(xy)$