CPS 102 **Homework 5** **Spring 2012**

Due Thursday, Mar. 22 at the beginning of class
46 points

1. (3 pts) Determine whether each of these numbers is prime. If it is a prime give the list of numbers you need to try to divide it by in determining whether or not it is a prime. If it is not a prime then give a prime factorization of it.

   (a) 19

   (b) 93

   (c) 107

2. (2 pts) Which positive integers less than 12 are relatively prime to 12?

3. (4 pts) Determine whether the integers in each of these sets are pairwise relatively prime. Explain.

   (a) 25, 41, 49, 64

   (b) 17, 18, 19, 23

4. (2 pts) What is the greatest common divisor of these integers?
   $2^2 \cdot 3^3 \cdot 5^5$, $2^5 \cdot 3^3 \cdot 5^2$

5. (2 pts) What is the least common multiple of these integers?
   $2^2 \cdot 3^3 \cdot 5^5$, $2^5 \cdot 3^3 \cdot 5^2$

6. (4 pts) Use the Euclidean algorithm to find

   (a) gcd(100,101)

   (b) gcd(1529, 14039)

7. (6 pts) Express the gcd of each of these pairs of integers as a linear combination of these integers.

   (a) 35,78

   (b) 124, 323

8. (3 pts) Find an inverse of 34 modulo 89 (note that 34 and 89 are relatively prime) by first using the Euclidean algorithm to show the gcd of the two is 1, then reverse the steps to find the Bezout coefficients a and b such that $34a + 89b = 1$. Then a would be the inverse of 34 modulo 89.

9. (3 pts) Solve the following congruence using the inverse found in the previous problem.

   $34x = 77 (\text{mod } 89)$

10. (3 pts) Solve the following system of congruences using the method of back substitution.

    $x \equiv 3(\text{ mod } 6)$, $x \equiv 4(\text{ mod } 7)$

11. (2 pts) Which memory locations are assigned by the hashing function $h(k) = k$ mod 101 to the records of insurance company customers with these Social Security numbers?

    (a) 104578690

    (b) 432222187

12. (4 pts) The United States Postal Service (USPS) sells money orders identified by an 11-digit number $x_1x_2x_3 \ldots x_{11}$. The first ten digits identify the money order; x_{11} is a check digit that satisfies $x_{11} = x_1 + x_2 + \ldots x_{10}$ mod 9.

    One digit in each of these identification numbers of a postal money order is smudged. Can you recover the smudged digit indicated by Q, in each of these numbers?

    (a) 27Q41007734

    (b) 213279032Q1

13. (3 pts) Encrypt the message "STOP POLLUTION" by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters. Use $f(p) = (p + 21)$ mod 26.

14. (2 pts) Decrypt the following message that was encrypted using the Caesar cipher.

    WHVW WRGDB

15. (3 pts) Encrypt the message "GRIZZLY BEARS" using blocks of five letters and the transposition cipher based on the permutation of 1,2,3,4,5 with $\sigma(1) = 3$, $\sigma(2) = 5$, $\sigma(3) = 1$, $\sigma(4) = 2$, and $\sigma(5) = 4$. For this exercise, use the letter X as many times as neccessary to fill out the final block of fewer than five letters.