CPS 102 **Homework 6** **Spring 2012**

Due Thursday, Mar. 29 at the beginning of class
35 points

1. (3 pts) Prove by mathematical induction that $1^3 + 2^3 + \ldots + n^3 = (n(n+1)/2)^2$ for the positive integer n.

2. (3 pts) Prove that $1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! = (n+1)! - 1$ whenever n is a positive integer.

3. (3 pts) Prove that for every positive integer n,

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \ldots + n(n+1)(n+2) = n(n+1)(n+2)(n+3)/4$$

4. (3 pts) Prove that $3^n < n!$ if n is an integer greater than 6.

5. (3 pts) Prove that 3 divides $n^3 + 2n$ whenever n is a positive integer.

6. (3 pts) Prove that if $A_1, A_2, \ldots A_n$ and B are sets, then $(A_1 - B) \cap (A_2 - B) \cap \ldots \cap (A_n - B) = (A_1 \cap A_2 \cap \ldots A_n) - B$

7. (5 pts) Let $P(n)$ be the statement that a postage of n cents can be formed using just 4-cent and 7-cent stamps. The parts of this exercise outline a strong induction proof that $P(n)$ is true for $n \geq 18$.

   (a) Show statements P(18), P(19), P(20) and P(21) are true, completing the basis step of the proof.
   (b) What is the inductive hypothesis of the proof?
   (c) What do you need to prove in the inductive step?
   (d) Complete the inductive step for $k \geq 21$.
   (e) Explain why these steps show that this statement is true whenever $n \geq 18$.

8. (3 pts) Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1, 2^1 = 2, 2^2 = 4$, and so on.

   Hint: For the inductive step, separately consider the case where $k+1$ is even and where it is odd. When it is even, note that $(k+1)/2$ is an integer.

9. (3 pts) For $f(n+1) = f(n)f(n-1)$, find $f(2), f(3), f(4)$, and $f(5)$ if f is defined recursively by $f(0) = f(1) = 1$, for $n = 1, 2, \ldots$.

10. (3 pts) Give a recursive definition of the set of positive integer powers of 3.

11. (3 pts) Recursively define the set of bit strings that have more zeros than ones.