Due Thursday, Apr 5 at the beginning of class
47 points

1. (2 pts) A particular brand of shirt comes in 12 colors, has a male and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?

2. (6 pts) How many strings of four decimal digits

   (a) do not contain the same digit twice?

   (b) end with an even digit?

   (c) have exactly three digits that are 9's?

3. (8 pts) How many strings of eight uppercase letters are there

   (a) if letters can be repeated?

   (b) if no letter can be repeated?

   (c) that start with X, if letters can be repeated?

   (d) that start with X, if no letter can be repeated?

4. (6 pts) How many ways can a photographer at a wedding arrange 6 people in a row from a group of ten people, where the bride and groom are among these 10 people, if

   (a) the bride must be in the picture?

   (b) both the bride and groom must be in the picture?

   (c) exactly one of the bride and the groom is in the picture?

5. (2 pts) There are six different candidates for govenor of a state. In how many different orders can the names of the candidates be printed on a ballot?

6. (8 pts) How many bit strings of length 12 contain

   (a) exactly three 1's?

   (b) at most three 1's?

   (c) at least three 1's?

   (d) an equal number of 0's and 1's?

7. (2 pts) How many ways are there for ten women and six men to stand in a line so that no two men stand next to each other? [Hint: First position the women, then the men.]

8. (6 pts) Thirteen people on a softball team show up for a game.

(a) How many ways are there to choose 10 players to take the field?

(b) How many ways are there to assign the 10 positions by selecting players from the 13 players who show up?

(c) Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?

9. (2 pts) What is the coefficient of x^7 in $(1+x)^{11}$?

10. (2 pts) The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}$, $0 \leq k \leq 10$ is:

    1 10 45 120 210 252 210 120 45 10 1

    Use Pascal's identity to produce the row immediately following this row in Pascal's triangle.

11. (3 pts) Suppose that k and n are integers with $1 \leq k < n$. Prove the identity

$$\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \binom{n-1}{k}\binom{n}{k-1}\binom{n+1}{k+1}$$