CPS 102 **Homework 8** **Spring 2012**

Due Thursday, Apr 19 at the beginning of class
45 points

1. (9 pts) What is the probability of these events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?

   (a) a is the first letter of the permutation and z is the last letter.

   (b) a and b are not next to each other in the permutation.

   (c) z precedes both a and b in the permutation.

2. (4 pts) Show that if E and F are events then $p(E \cap F) \geq p(E) + p(F) - 1$.

3. (4 pts) Use mathematical induction to prove the following.
   $p(E_1 \cap E_2 \cap \ldots \cap E_n) \geq p(E_1) + p(E_2) + \ldots + p(E_n) - (n-1)$
   where $E_1, E_2, \ldots E_n$ are n events.

4. (3 pts) Let E be the event that a randomly generated bit string of length three contains an odd number of 1s and let F be the event that the string starts with 1. Are E and F independent?

5. (9 pts) Find the probability that a randomly generated bit string of length 10 does not contain a 0 if bits are independent and if

   (a) a 0 bit and a 1 bit are equally likely

   (b) the probability that a bit is a 1 is 0.6

   (c) the probability that the ith bit is a 1 is $1/2^i$ for $i = 1, 2, 3, \ldots 10$.

6. (4 pts) When a test for steroids is given to soccer players, 98% of the players taking steroids test positive, and 12% of the players not taking steroids test positive. Suppose that 5% of soccer players take steroids. What is the probability that a soccer player who tests positive takes steroids? (HINT: Use Bayes Theorem)

7. (3 pts) A coin is biased so that the probability that a head comes up when it is flipped is 0.6. What is the expected number of heads that come up when it is flipped 10 times?

8. (6 pts) Suppose that we roll a fair die until a six comes up.

   (a) What is the probability that we roll the die n times?

   (b) What is the expected number of times we roll the die?

9. (3 pts) What is the probability that a randomly selected bit string of length 11 is a palindrome?