CPS 102 **Homework 9** **Spring 2012**

Due Tuesday, Apr 24 at the beginning of class
30 points

1. (5 pts)
   (a) Let a_n be the number of bit strings of length n that contain three consecutive 0s. Find a recurrence relation for a_n.
   (b) What are the initial conditions?
   (c) How many bit strings of length seven contain three consecutive 0's?

2. (6 pts) Solve these recurrence relations with the initial conditions given.
   (a) $a_n = 6a_{n-1} - 8a_{n-2}$ for $n \geq 2$, $a_0 = 4, a_1 = 10$
   (b) $a_n = -6a_{n-1} - 9a_{n-2}$ for $n \geq 2$, $a_0 = 3, a_1 = -3$

3. (6 pts)
   (a) Find all solutions of the recurrence $a_n = -5a_{n-1} - 6a_{n-2} + 42 * 4^n$.
   (b) Find the solution of this recurrence relation with $a_1 = 56$ and $a_2 = 278$.

4. (3 pts) Find $f(n)$ when $n = 3^k$, where f satisfies the recurrence relation $f(n) = 2f(n/3) + 4$ with $f(1) = 1$. (Hint: use Theorem 1 from Section 8.3).

5. (4 pts) A simple graph is a graph in which each edge connects two different vertices and no two edges connect the same pair of vertices. Show that in a simple graph with at least two nodes, there must be two nodes that have the same degree.

6. (6 pts) Let G be a graph with n nodes and e edges. Let M be the maximum degree of the nodes and m be the minimum degree of the nodes.
   (a) Show $2e/n \geq m$
   (b) Show $2e/n \leq M$