CompSci 102
Discrete Math for Computer Science

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

January 17, 2012

Prof. Rodger

Announcements

- Read for next time Chap. 1.1-1.3
- Recitations start Friday
- Added everyone to Piazza
- ACM Distinguished Speaker tonight
  - Dilma Da Silva, IBM
  - System Software for Cloud Computing
  - 6pm tonight in LSRC D106 with Pizza!

Logic

- Rules of logic specify meaning of mathematical statements
- How do you understand:
- $\forall n > 0 \ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- Applications in CS
  - Designing computers
  - Designing programming languages
  - Correctness of programs
  - Many areas such as artificial intelligence

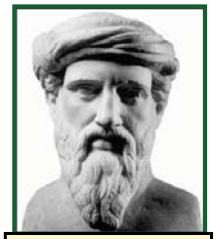
Proposition

- A **proposition** is a sentence that declares a fact that is true or false
- A **theorem** is a proposition that is guaranteed by a proof

Examples of Propositions

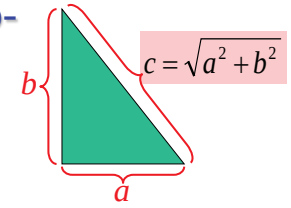
- Which are propositions? What is their value?
  1. Duke won the NCAA men's basketball title in 2010.
  2. 3x > 2
  3. Clean up after yourself.
  4. Durham is the capital of NC.
  5. Pepsi was invented in New Bern NC in 1898.
  6. 8 + 3 = 11

A Proof Example



Pythagoras of Samos (ca. 569-475 B.C.)

- **Theorem:** *(Pythagorean Theorem of Euclidean geometry)* For any real numbers a, b, and c, if a and b are the base-length and height of a right triangle, and c is the length of its hypotenuse, then $a^2 + b^2 = c^2$.
- **Proof?**



Operators / Connectives

An *operator* or *connective* combines one or more *operand* expressions into a larger expression. (*E.g.*, "+" in numeric exprs.)

- *Unary* operators take 1 operand (*e.g.*, −3); *binary* operators take 2 operands (*eg* 3 × 4).
- *Propositional* or *Boolean* operators operate on propositions (or their truth values) instead of on numbers.

The Negation Operator

The unary *negation operator* "¬" (*NOT*) transforms a prop. into its logical *negation*.

E.g. If p = "I have brown hair."

then $\neg p$ = "I do **not** have brown hair."

The *truth table* for NOT:

p	$\neg p$
T	F
F	T

Operand column | Result column

T :≡ True; F :≡ False

":≡" means "is defined as"

Conjunction Truth Table

- A conjunction $p_1 \wedge p_2 \wedge \ldots \wedge p_n$ of n propositions will have how many rows in its truth table?
- Note: ¬ and ∧ operations together are suffi-cient to express *any* Boolean truth table!

Operand columns

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Disjunction Truth Table

- Note that $p \vee q$ means that p is true, or q is true, **or both** are true!
- So, this operation is also called *inclusive or*, because it **includes** the possibility that both p and q are true.
- "¬" and "∨" together are also universal.

p	q	$p \vee q$
F	F	F
F	T	**T**
T	F	**T**
T	T	T

Note difference from AND

Exclusive-Or Truth Table

- Note that $p \oplus q$ means that p is true, or q is true, but **not both**!
- This operation is called *exclusive or*, because it **excludes** the possibility that both p and q are true.
- "¬" and "⊕" together are **not** universal.

p	q	$p \oplus q$
F	F	F
F	T	T
T	F	T
T	T	**F**

Note difference from OR.

Implication Truth Table

- $p \rightarrow q$ is **false** only when p is true but q is **not** true.
- $p \rightarrow q$ does **not** say that p causes q!
- $p \rightarrow q$ does **not** require that p or q **are ever true**!
- *E.g.* "(1=0) → pigs can fly" is TRUE!

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	**F**
T	T	T

The only False case!

English Phrases Meaning $p \rightarrow q$

- "p implies q"
- "if p, then q"
- "if p, q"
- "when p, q"
- "whenever p, q"
- "q if p"
- "q when p"
- "q whenever p"
- "p only if q"
- "p is sufficient for q"
- "q is necessary for p"
- "q follows from p"
- "q is implied by p"

We will see some equivalent logic expressions later.

Converse, Inverse, Contrapositive

Some terminology, for an implication $p \rightarrow q$:

- Its *converse* is: $q \rightarrow p$.
- Its *inverse* is: $\neg p \rightarrow \neg q$.
- Its *contrapositive*: $\neg q \rightarrow \neg p$.
- One of these three has the *same meaning* (same truth table) as $p \rightarrow q$. Can you figure out which?

Biconditional Truth Table

- $p \leftrightarrow q$ means that p and q have the **same** truth value.
- Note this truth table is the exact **opposite** of $\oplus$'s!

  Thus, $p \leftrightarrow q$ means $\neg(p \oplus q)$
- $p \leftrightarrow q$ does **not** imply that p and q are true, or that either of them causes the other, or that they have a common cause.

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

Boolean Operations Summary

- We have seen 1 unary operator and 5 binary operators. Their truth tables are below.

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$
F	F	T	F	F	F	T	T
F	T	T	F	T	T	T	F
T	F	F	F	T	T	F	F
T	T	F	T	T	F	T	T