CompSci 102
Discrete Math for Computer Science

Feb. 2, 2012

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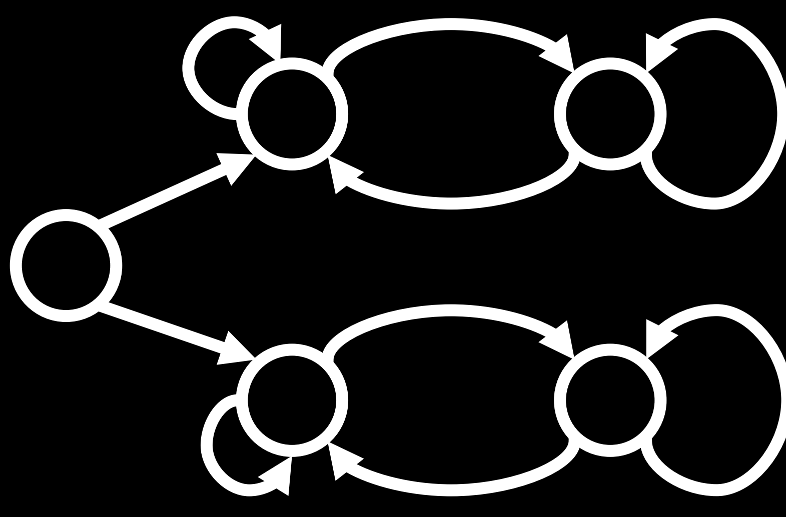
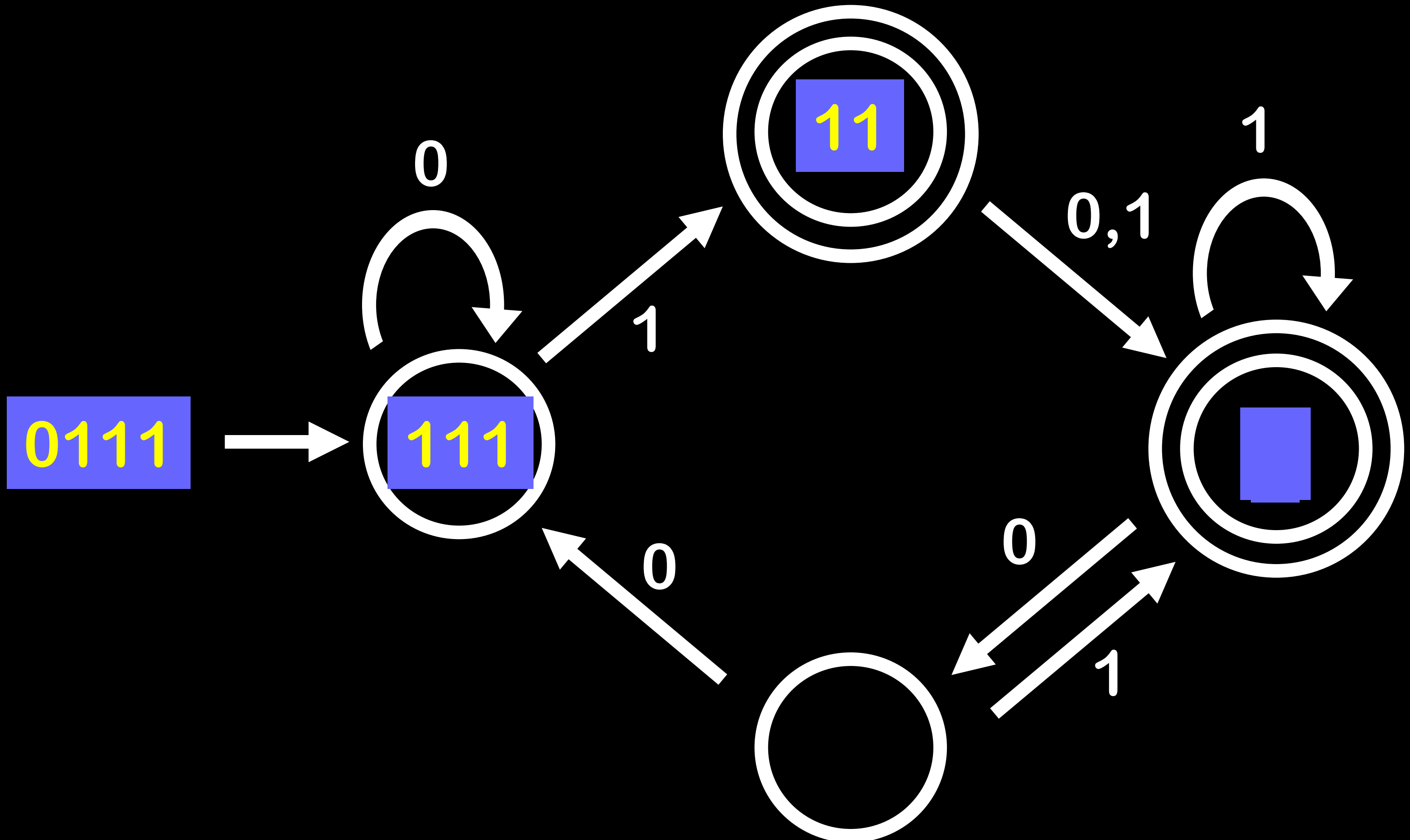
Slides from Maggs, lecture developed by Steven Rudich at CMU

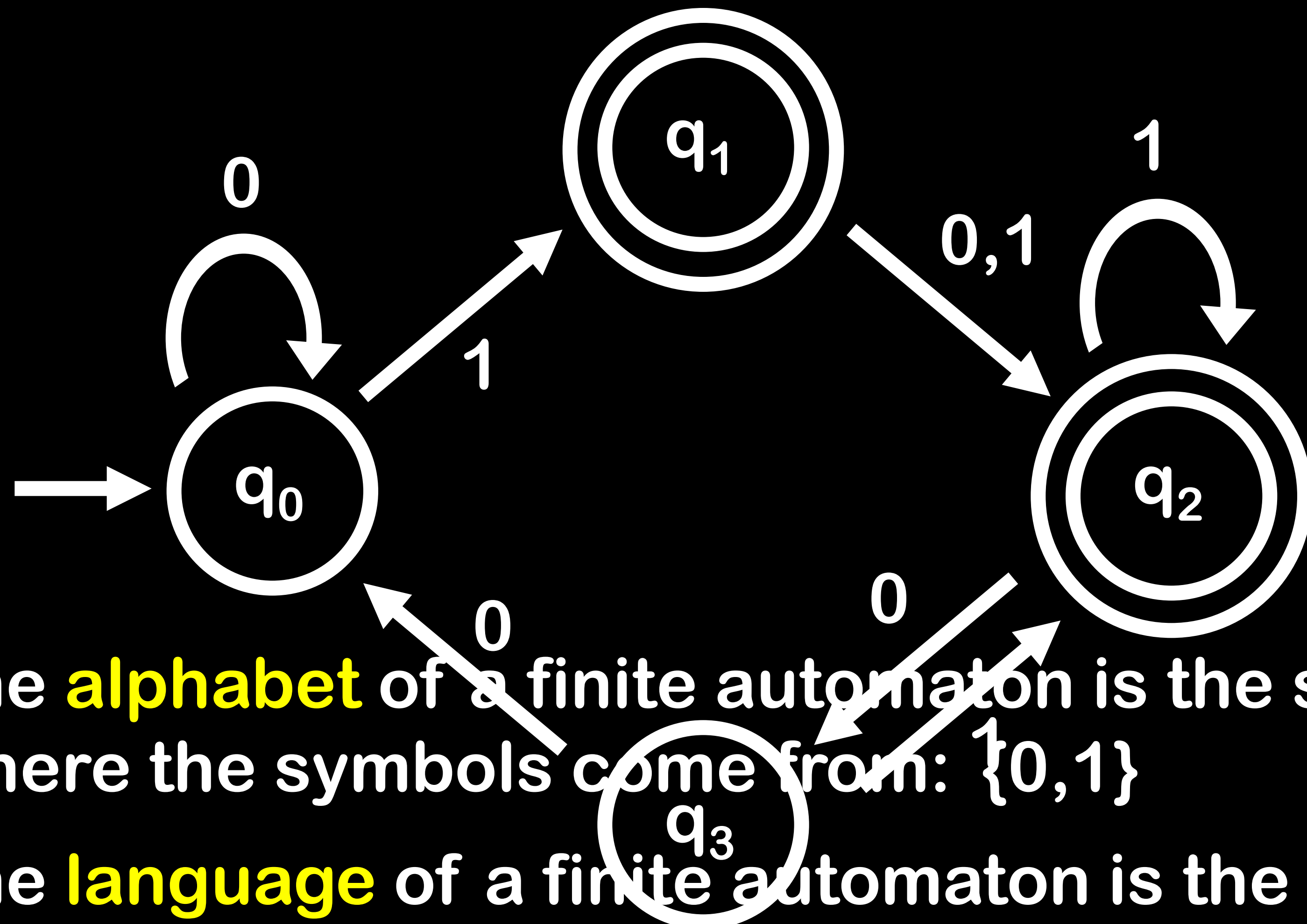
Deterministic Finite Automata





The machine **accepts** a string if the process ends in a double circle

Anatomy of a Deterministic Finite Automaton



The **alphabet** of a finite automaton is the set where the symbols come from: {0,1}

The **language** of a finite automaton is the set of strings that it accepts

Notation

An alphabet Σ is a finite set (e.g., $\Sigma = \{0,1\}$)

A string over Σ is a finite-length sequence of elements of Σ

For a string x, $|x|$ is the length of x

The unique string of length 0 will be denoted by ε and will be called the empty or null string

A language over Σ is a set of strings over Σ

A finite automaton is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$

Q is the set of states

Σ is the alphabet

$\delta : Q \times \Sigma \rightarrow Q$ is the transition function

$q_0 \in Q$ is the start state

$F \subseteq Q$ is the set of accept states

L(M) = the language of machine M
= set of all strings machine M accepts

A language is regular if it is recognized (accepted) by a deterministic finite automaton

L = { w | w contains 001} is regular

L = { w | w has an even number of 1s} is regular

Union Theorem

Given two languages, L_1 and L_2, define the union of L_1 and L_2 as

$$L_1 \cup L_2 = \{ w \mid w \in L_1 \text{ or } w \in L_2 \}$$

Theorem: The union of two regular languages is also a regular language

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Proof Sketch: Let

$M_1 = (Q_1, \Sigma, \delta_1, q_0^1, F_1)$ be finite automaton for L_1

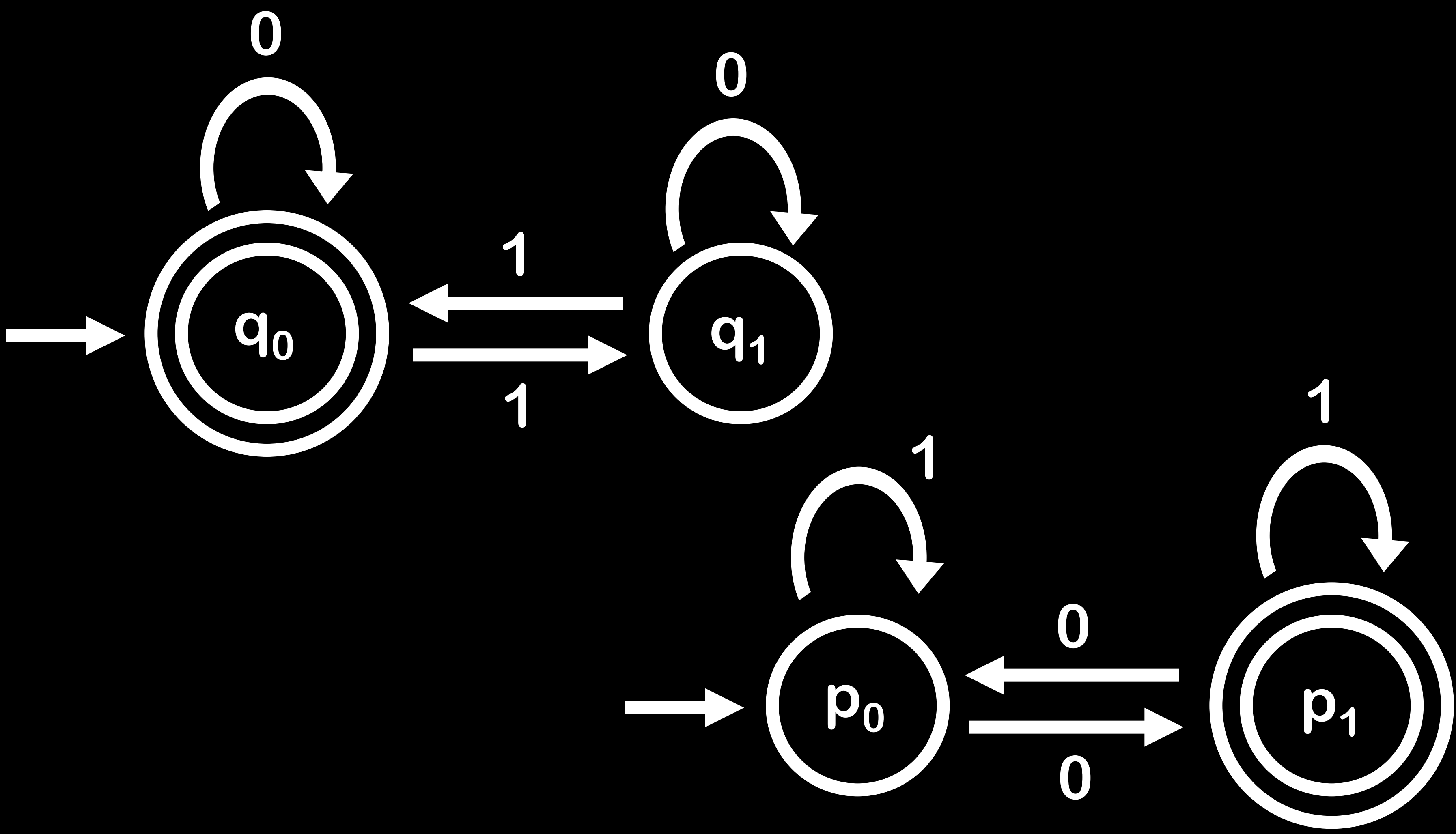
and

$M_2 = (Q_2, \Sigma, \delta_2, q_0^2, F_2)$ be finite automaton for L_2

We want to construct a finite automaton

$M = (Q, \Sigma, \delta, q_0, F)$ that recognizes $L = L_1 \cup L_2$

Theorem: The union of two regular languages is also a regular language



Theorem: The union of two regular languages is also a regular language

Corollary: Any finite language is regular

The Regular Operations

Union: $A \cup B = \{ w \mid w \in A \text{ or } w \in B \}$

Intersection: $A \cap B = \{ w \mid w \in A \text{ and } w \in B \}$

Reverse: $A^R = \{ w_1 \ldots w_k \mid w_k \ldots w_1 \in A \}$

Negation: $\neg A = \{ w \mid w \notin A \}$

Concatenation: $A \cdot B = \{ vw \mid v \in A \text{ and } w \in B \}$

Star: $A^* = \{ w_1 \ldots w_k \mid k \geq 0 \text{ and each } w_i \in A \}$

Regular Languages Are Closed Under The Regular Operations

We have seen part of the proof for Union. The proof for intersection is very similar. The proof for negation is easy.

The "Grep" Problem

Input: Text T of length t, string S of length n

Problem: Does string S appear inside text T?

Naïve method:

$$\boxed{a_1, a_2, a_3,}\, a_4, a_5, \ldots, a_t$$

Cost: Roughly nt comparisons

Automata Solution

Build a machine M that accepts any string with S as a consecutive substring

Feed the text to M

Cost: t comparisons + time to build M

As luck would have it, the Knuth, Morris, Pratt algorithm builds M quickly

Real-life Uses of DFAs

Grep

Coke Machines

Thermostats (fridge)

Elevators

Train Track Switches

Lexical Analyzers for Parsers

Consider the language $L = \{ a^n b^n \mid n > 0 \}$

i.e., a bunch of a’s followed by an equal number of b’s

No finite automaton accepts this language

Can you prove this?

Pigeonhole principle:

Given n boxes and m > n objects, at least one box must contain more than one object

Letterbox principle:

If the average number of letters per box is x, then some box will have at least x letters (similarly, some box has at most x)

Theorem: L= $\{a^n b^n \mid n > 0\}$ is not regular

Proof (by contradiction):

Assume that L is regular

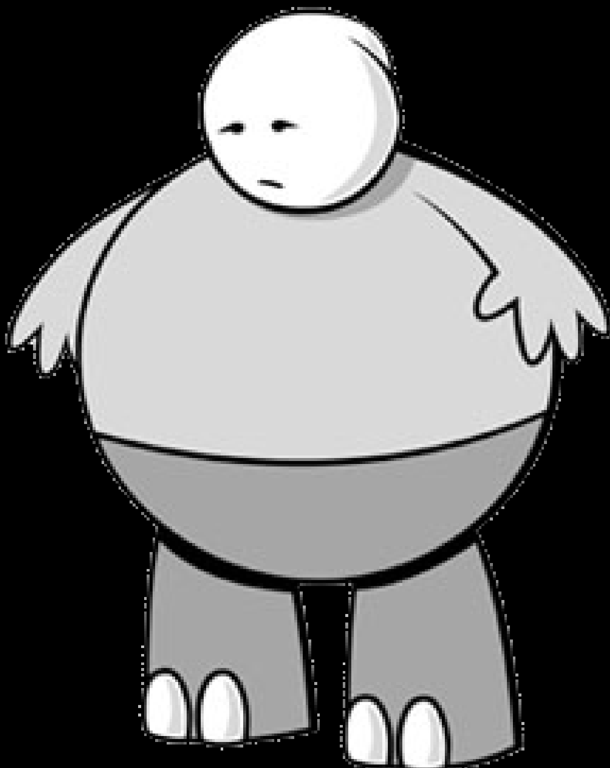
Then there exists a machine M with k states that accepts L

For each $0 \leq i \leq k$, let S_i be the state M is in after reading a^i

$\exists i,j \leq k$ such that $S_i = S_j$, but $i \neq j$

M will do the same thing on $a^i b^i$ and $a^j b^i$

But a valid M must reject $a^j b^i$ and accept $a^i b^i$



Here's What You Need to Know…

Deterministic Finite Automata

- Definition
- Testing if they accept a string
- Building automata

Regular Languages

- Definition
- Closed Under Union, Intersection, Negation
- Using Pigeonhole Principle to show language ain't regular