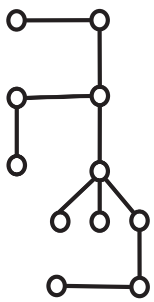
CompSci 102
Discrete Math for Computer Science



April 19, 2012

Prof. Rodger

Lecture adapted from Bruce Maggs/Lecture developed at Carnegie Mellon, primarily by Prof. Steven Rudich.

Announcements

- Read for next time Chap 10.3-4
- Last homework out, due Tuesday
- No Recitation on FRIDAY this week after all
- Graphs

Approximating Graphs and Solving Systems of Linear Equations

Speaker:	Daniel Spielman Yale University
Date:	Monday, April 23, 2012
Time:	4:00pm - 5:00pm
Location:	130A North Building, Duke

Abstract

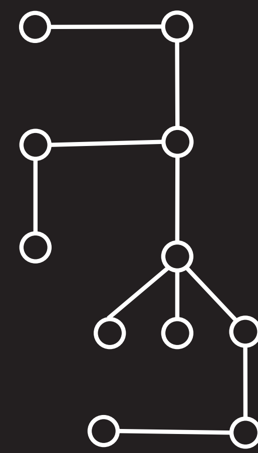
We survey recent advances in the design of algorithms for solving linear equations in the Laplacian matrices of graphs. These algorithms motivate and rely upon fascinating concepts in graph theory, the most important of which is a definition of what it means for one graph to approximate another. This definition leads to the problem of sparsification: the approximation of one graph by a sparser graph.

The sparsest possible approximation of a graph is a spanning tree. The average stretch of a spanning tree will be revealed to be a good measure of its approximation quality. We explain how every graph on n vertices can be well-approximated by a graph with O (n) edges. To solve linear equations, we will require something in between: approximations by graphs with n + O (n / log n) vertices. To build these sparse approximations, we employ low-stretch spanning trees, random matrix theory, spectral graph theory, and graph partitioning algorithms.

Graphs

What’s a tree?

Tree



Notation

In this lecture:

n will denote the number of nodes in a graph

e will denote the number of edges in a graph

Theorem: Let G be a graph with n nodes and e edges

The following are equivalent:

1. G is a tree (connected, acyclic)
2. Every two nodes of G are joined by a unique path
3. G is connected and $n = e + 1$
4. G is acyclic and $n = e + 1$
5. G is acyclic and if any two non-adjacent points are joined by adding a new edge, the resulting graph has exactly one cycle

To prove this, it suffices to show

$$1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 4 \Rightarrow 5 \Rightarrow 1$$

How to reconstruct the unique tree from a sequence S:

Let I = {1, 2, 3, …, n}

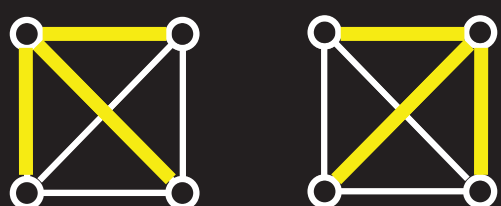
Loop until S is empty

- Let i = smallest # in I but not in S
- Let s = first label in sequence S
  - Add edge {i, s} to the tree
  - Delete i from I
  - Delete s from S

Add edge {a,b}, where I = {a,b}

Spanning Trees

A spanning tree of a graph G is a tree that touches every node of G and uses only edges from G



Every connected graph has a spanning tree

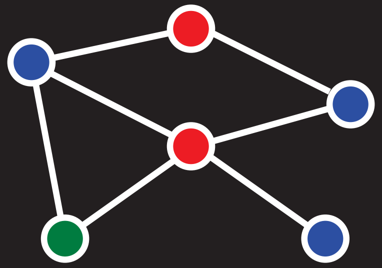
A graph is planar if it can be drawn in the plane without crossing edges

Euler's Formula

If G is a connected planar graph with n vertices, e edges and f faces, then $n - e + f = 2$

Graph Coloring

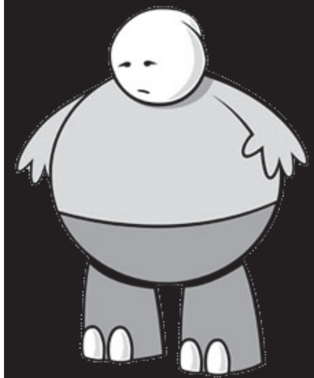
A coloring of a graph is an assignment of a color to each vertex such that no neighboring vertices have the same color



Graph Coloring

Arises surprisingly often in CS

Register allocation: assign temporary variables to registers for scheduling instructions. Variables that interfere, or are simultaneously active, cannot be assigned to the same register



Here's What You Need to Know…

Trees

- Counting Trees
- Different Characterizations

Planar Graphs

- Definition
- Euler's Theorem
- Coloring Planar Graphs

Adjacency Matrix and List

- Definition
- Useful for counting