CPS 102 EXAM 1 SOLUTIONS Feb. 2012

1. a) $(p \vee q) \rightarrow (p \wedge q)$

$(F \vee F) \rightarrow (F \wedge F)$

$F \rightarrow F$

TRUE

1. b) $(p \vee q) \rightarrow (p \wedge q)$

$(T \vee T) \rightarrow (T \wedge T)$

$T \rightarrow T$

TRUE

2. $\neg(p \vee \neg q)$

$= \neg p \wedge \neg\neg q$ by De Morgan's laws

$= \neg p \wedge q$ by double negation

$= q \wedge \neg p$ by Commutative law

3 a) For every $n \geq 0$ there is an integer m such that $n < m$ (for example $m = n+1$).
This is false.

b) For every integer m there is an integer n s.t. $n \geq m$ ($n = m+1$).
This is true.

4. A is a Knight and B is a Knave.

Suppose A is telling the truth. Then A is a Knight. That means B must be a Knave. B says "A is a Knave" and is lying so he is a Knave.
Thus A could be a Knight & B a Knave.

Suppose A is lying. Then A would be a Knave. But his statement would have to be a lie $\Rightarrow$ No one is a Knave.
Thus A must be telling the truth, so A is a Knight & B is a Knave.

5.

1.	$\forall x (P(x) \rightarrow Q(x))$	given
2.	$\forall x (Q(x) \rightarrow R(x))$	given
3.	$\neg R(a)$	given
4.	$P(a) \rightarrow Q(a)$	by 1, & Univ. Instan.
5.	$Q(a) \rightarrow R(a)$	by 2. & Universal Instantiate
6.	$P(a) \rightarrow R(a)$	by 4,5 & Hypoth. syllogism
7.	$\neg P(a)$	by 3,6 & Modus tollens

6. a) $A \cap C = \{6\}$

b) $B - C = \{1, 2, 4\}$

c) $|B \times A| = 16$

d) $C \times \{2\} = \{(3,2), (6,2)\}$

e) False

f) $\sum_{j=1}^{100} (-1)^j =$ $-1+1$ 50 times $= 0$

6f alternate

$\left[\sum_{K=0}^{100} 1 \cdot (-1)^K\right] - 1 = \frac{(-1)^{101} - 1}{-1 - 1} - 1$

$= -1 - 1$

$= 1 - 1 = 0$

7. Let $A = \{1,2,3\}$, $B = \{2\}$, $C = \{3\}$

$A - (B \cap C) = \{1,2,3\} - \{\} = \{1,2,3\}$

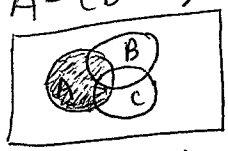
$(A-B) \cap (A-C) = \{1,3\} \cap \{1,2\} = \{1\}$

$\{1,2,3\} \neq \{1\}$

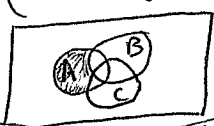
False

7. alt. Venn Diagram

$A - (B \cap C) =$



$(A-B) \cap (A-C) =$



False

8a. If $f(n) = f(m)$ then $2n+1 = 2m+1$,

$\Rightarrow n = m$. Hence f is one-to-one.

b. $f(n) = 2n+1$ is odd for every integer n.

Thus 2 is not in its range.

f is not onto.

9. $1^3 + 2^3 + 3^3 + \ldots + n^3$

$\leq n^3 + n^3 + n^3 + \ldots + n^3 = n^4$ for $n > 0$.

$C = 1$, $K = 1$.

$|f(n)| \leq n^4$ for $n > 1$

10. a) $O(x^3)$

b) $O(x^2)$

11. a) $(2D)_{16} = 2 \times 16 + 13 = 32 + 13 = 45$

b) $39 = 32 + 4 + 2 + 1$

$= 1 \cdot 2^5 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$

$= 100111$

c)
$$\begin{array}{r} {}^{1}(212)_3 \\ +\,(102)_3 \\ \hline (1021)_3 \end{array}$$

$2 \cdot 3^2 + 1 \cdot 3^1 + 2 \cdot 3^0$

$18 + 3 + 2 = 23$

$1 \cdot 3^2 + 2 \cdot 3^0 = +\,11$

$(34)_{10}$ ✓

$1 \cdot 3^3 + 2 \cdot 3^1 + 1 \cdot 3^0$

$27 + 6 + 1 = (34)_{10}$

12 a) $(32 \bmod 7) = 4$

$(4 \bmod 3) = 1$

b) $x \equiv 22 \pmod 8$

$x = 30$ $\quad 30 - 22 = 8$ $\quad$ 8 divides 8 ✓

$x = 38$ $\quad 38 - 22 = 16$ $\quad$ 8 divides 16 ✓

13 a) $\{q_0, q_1, q_2, q_3\}$ ← with trap state would be $\{q_0, q_1, q_2, q_3, q_4\}$

b) $\{q_3\}$

c) aba, baaba, abbaba

d) Strings of a's & b's that end with aba, and do not have aba anywhere except at the end of the string.

14.