CPS 102 **Recitation 2** **Spring 2012**

Friday, Jan. 27

1. Let C(x) be the statement "x has a cat", D(x) be the statement "x has a dog", and let F(x) be the statement "x has a ferret". Express each of these statements in terms of C(x), D(x), F(x), quantifiers, and logical connectives. Let the domain consist of all students in your class.

   (a) All students in your class have a cat, a dog or a ferret.

   (b) For each of the three animals, cats, dogs and ferrets, there is a student in your class who has this animal as a pet.

2. Let F(x,y) be the statement "x can fool y", where the domain consists of all people in the world. Use quantifiers to exress each of these statements.

   (a) Evelyn can fool everybody.

   (b) Everyone can be fooled by somebody.

   (c) Everybody can fool somebody.

3. For each of the following, define proposition symbols for each simple proposition in the argument (for example, $P =$ "the pollution is dangerous"). Then write out the logical form of the argument. If the argument form corresponds to a known inference rule, say which it is. If not, show that the proof is correct using truth tables.

   (a) It is hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees today. Therefore, the pollution is dangerous.

   (b) Steve will work at a computer company this summer. Therefore, Steve will work at a computer company this summer or he will be a beach bum.

   (c) If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on. If the sailing race is held, then the trophy will be awarded. The trophy was not awarded. Therefore, it rained.

4. Truth Tables

   In class and in the book you saw several truth tables that take two propositions as their input. The operators associated to these tables are called *binary*, because they take two propositions as operands. For instance,

p	q	
T	T	T
T	F	F
F	T	F
F	F	F

is the truth table for the conjunction ("and") of the two propositions p and q, that is, the truth table of $p \wedge q$, so conjunction is a binary operator. How many distinct binary truth tables could one conceivably make up? The answer is sixteen: Truth tables for binary operators all have four rows, and the tables differ from one another by the content of the four elements in the last column. There are sixteen possible ways to fill the last column, and here they are:

p	q	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	r
T	T	T	T	T	T	T	T	T	T	F	F	F	F	F	F	F	F
T	F	T	T	T	T	F	F	F	F	T	T	T	T	F	F	F	F
F	T	T	T	F	F	T	T	F	F	T	T	F	F	T	T	F	F
F	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F

Some of these are familiar: `a` is a tautology, `r` is a contradiction, `b` is $p \vee q$, `c` is $q \to p$, `e` is $p \to q$, and so forth. Some others we have never seen before.

Table `i` is called the *nand* of p and q. Let us use a special symbol for it, $p \,|\, q$ (called *Sheffer's stroke*). Let us repeat its truth table here:

p	q	$p \,\vert\, q$
T	T	F
T	F	T
F	T	T
F	F	T

.

From this table, we can see immediately the following logical equivalence:

$$p \,|\, q \equiv \neg(p \wedge q) \ ,$$

so we can obtain the table for "and" (column `h`) from that of "nand" by inverting the equivalence:

$$p \wedge q \equiv \neg(p \,|\, q) \ .$$

Write sixteen lines in the following format:

`x` : *expression*

where `x` represents one of the sixteen letters in the column headings of the table above, and *expression* is an implementation of the corresponding truth table using nothing other than the symbols p, q, "nand" operators and parentheses. For example,

`i` : $p \,|\, q$.

There may be different answers for each column. Just give any one. Show your reasoning, and also give the final table neatly written with the columns in alphabetical order. [Hint: Column `m` is the truth table for $\neg p$, which can be implemented as follows:

`m` : $p \,|\, p$

(check with a two-row truth table that this is the case). It is best to build your answer from the easiest cases, and derive harder answers from easier ones.]