CPS 102 **Recitation 4** **Spring 2012**

Friday, Feb. 10

1. Let A and B be sets. Show that $A \cap (B - A) = \emptyset$
2. Find the domain and range of the function that assigns to each positive integer its largest decimal digit.
3. Determine whether $f : \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$ is onto if $f(m,n) = m^2 - n^2$.
4. Determine whether $f(x) = -3x^2 + 7$ is a bijection from $\mathbf{R}$ to $\mathbf{R}$.
5. Show that if x is a real number and m is an integer, then $\lceil x + m \rceil = \lceil x \rceil + m$
6. Prove or disprove the following statement. $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$ for all real numbers x and y.
7. For the following sequence, find a recurrence relation satisfied by the sequence

   $a_n = n^2 + n$
8. Find the solution to the recurrence relation with the given initial condition. Use an iterative approach.

   $a_n = 2a_{n-1} - 3$, $a_0 = -1$
9. Assume that the population of the world in 2010 was 6.9 billion and is growing at the rate of 1.1% a year.

   (a) Set up a recurrence relation for the population of the world n years from 2010.

   (b) Find an explicit formula for the population of the world n years after 2010.

   (c) What will the population of the world be in 2030?
10. For the following list of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula of rule is correct, determine the next three terms of the sequence.

    3, 6, 11, 18, 27, 38, 51, 66, 83, 102, ...
11. Use the definition of "$f(x)$ is $O(g(x))$" to show that

    $2^x + 17$ is $O(3^x)$
12. Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions.

    (a) $f(x) = 2x^2 + x^3 \log x$

    (b) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$

13. Show that x^3 is $O(x^4)$, but that x^4 is not $O(x^3)$
14. Suppose that $f(x)$, $g(x)$ and $h(x)$ are functions such that $f(x)$ is $\Theta(g(x))$ and $g(x)$ is $\Theta(h(x))$. Show that $f(x)$ is $\Theta(h(x))$.
15. Show that $x^5y^3 + x^4y^4 + x^3y^5$ is $\Omega(x^3y^3)$.