CPS 102 **Recitation 5** **Spring 2012**

Friday, Feb. 17

1. Let a, b, and c be integers, with $a \neq 0$. Prove that if $a|b$ and $b|c$ then $a|c$.
2. What are the quotient and remainder when -123 is divided by 19?
3. Suppose that a and b are integers. $a \equiv 11(\ \mathbf{mod}\ 19)$ and $b \equiv 3(\ \mathbf{mod}\ 19)$. Find the integer c with $0 \leq c \leq 18$ such that $c \equiv 7a + 3b(\ \mathbf{mod}\ 19)$.
4. List five integers that are congruent to 4 modulo 12.
5. Find the value of $(19^2\ \mathbf{mod}\ 41)\ \mathbf{mod}\ 9$.
6. Show that if a, b, c and m are integers such that $m \geq 2$, $c > 0$ and $a \equiv b(\ \mathbf{mod}\ m)$, then $ac \equiv bc(\ \mathbf{mod}\ mc)$.
7. Prove that if n is an odd positive integer, then $n^2 \equiv 1(\ \mathbf{mod}\ \ 8)$.
8. Convert the decimal number 321 to a binary number.
9. Convert the binary number $(11011)_2$ to a decimal number.
10. Convert the octal number $(1604)_8$ to a binary number.
11. Convert the binary number $(101010101010)_2$ to an octal number.
12. Convert the hexadecimal number $(135AB)_{16}$ to a binary number.
13. Explain how to convert from binary to base 64 expansions and from base 64 expansions to binary expansions and from octal to base 64 expansions and from base 64 expansions to octal expansions.
14. Find the sum and product of the following pair: $(2112)_3, (12021)_3$.
15. REVIEW: Let $\Sigma = \{a, b\}$. For each of the following languages, give the state diagram for a DFA that recognizes it.
    (a) $L = \{w :$ the number of b's in w is divisible by $3\}$
    (b) $L = \{w :$ the number of b's in w is divisible by 3 and aa is a substring of w $\}$