Friday, Mar. 16

1. Prove that for every positive integer n, there are n consecutive composite integers.
   [Hint: Consider the n consecutive integers starting with $(n+1)! + 2$]

2. Determine whether the integers in each of these sets are pairwise relatively prime. Explain.
   (a) 21, 34, 55
   (b) 14,17,85

3. What is the greatest common divisor of these integers?
   $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$, $2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$

4. What is the least common multiple of these integers?
   $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$, $2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$

5. Use the Euclidean algorithm to find gcd(1529, 14038).

6. Express the gcd(2002,2339) as a linear combination of 2002 and 2339.

7. Find an inverse of 144 modulo 233 (note that 144 and 233 are relatively prime) by first using the Euclidean algorithm to show the gcd of the two is 1, then reverse the steps to find the Bezout coefficients a and b such that $144a + 233b = 1$. Then a would be the inverse of 144 modulo 233.

8. Solve the following congruence using the inverse found in the previous problem.

   $144x = 4 (\text{mod } 233)$

9. Show that the positive integers less than 11, except 1 and 10, can be split into pairs of integers such that each pair consists of integers that are inverses of each other modulo 11.

10. Solve the following system of congruences using the method of back substitution.
$x \equiv 1(\mod 2)$, $x \equiv 2(\mod 3)$, $x \equiv 3(\mod 5)$, $x \equiv 4(\mod 11)$

11. Which integers are divisible by 5 but leave a remainder of 1 when divided by 3?

12. The United States Postal Service (USPS) sells money orders identified by an 11-digit number $x_1x_2x_3 \ldots x_{11}$. The first ten digits identify the money order; x_{11} is a check digit that satisfies $x_{11} = x_1 + x_2 + \ldots x_{10} \mod 9$. Find the check digit for the USPS money order that has the identification number that starts with these ten digits: 7555618873.

13. One digit in each of these identification numbers of a postal money order is smudged (see the information about USPS identification numbers in the previous problem). Can you recover the smudged digit indicated by Q, in each of these numbers?

    (a) Q1223139784

    (b) 6702120Q988

14. Encrypt the message "STOP POLLUTION" by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters. Use $f(p) = (p + 4) \bmod 26$.

15. Decrypt the following message that was encrypted using the Caesar cipher.

    EOXH MHDQV

16. Suppose that the following ciphertext was produced by encrypting a plaintext message using a shift cipher. What is the original plaintext?

    DVE CFMV KF NFEUVI, REU KYRK ZJ KYV JVVU FW JTZVETV