CPS 102 **Recitation 7** **Spring 2012**

Friday, Mar. 23

1. Use mathematical induction to prove the following. Prove that $2 - 2 \cdot 7 + 2 \cdot 7^2 - \ldots + 2 \cdot (-7)^n = (1 - (-7)^{n+1})/4$ whenever n is a nonnegative integer.

2. (a) Find a formula for $1/2 + 1/4 + 1/8 + \ldots + 1/2^n$ by examining the values of this expression for small values of n.

   (b) Prove the formula you conjectured.

3. Prove that for every positive integer n, $\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2$

4. For which nonnegative integers n is $n^2 \leq n!$? Prove your answer.

5. Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer.

6. Prove that if $A_1, A_2, \ldots, A_n$ and B are sets, then $(A_1 \cap A_2 \cap \ldots \cap A_n) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \ldots \cap (A_n \cup B)$

7. Prove or disprove that a checkerboard with shape 6×2^n can be completely covered using right triominoes whenever n is a positive integer.

8. (a) Determine which amounts of postage can be formed using just 3-cent and 10-cent stamps.
   (b) Prove your answer to a) using the principle of mathematical induction. Be sure to state explicitly your inductive hypothesis in the inductive step.
   (c) Prove your answer to a) using strong induction. How does the inductive hyposthesis in this proof differ from that in the inductive hypothesis for a proof using mathematical induction?

9. Prove that $f_1^2 + f_2^2 + \ldots + f_n^2 = f_n f_{n+1}$ where f_n is the nth Fibonacci number.

10. Give a recursive definition of the set of odd positive integers.

11. Let S be the subset of the set of ordered pairs of integers defined recursively by

Basis step: (0,0) $\in$ S

Recursive step: If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$

(a) List the elements of S produced by the first five applications of the recursive definition.

(b) Use strong induction on the number of applications of the recursive step of the definition to show that $5 \mid a + b$ when $(a, b) \in S$.

(c) Use structural induction to show that $5 \mid a + b$ when $(a, b) \in S$.

12. Give a recursive definition of the set of bit strings that are palindromes.

13. Play the game Chomp at this website.

    http://www.math.ucla.edu/∼tom/Games/chomp.html

    Is there a winning strategy for the player who goes first?

14. Prove that the first player has a winning strategy for the game of Chomp if the initial board is two squares wide, that is a $2 \times n$ board. [Hint: Use strong induction.]