

Friday, Mar. 30

1. How many bit strings of length n , where n is a positive integer, start and end with 1's?
2. A DNA sequence is a sequence of letters, each of which is one of A, C, G, or T. How many 5 element DNA sequences
 - (a) end with A?
 - (b) start with T and end with G?
 - (c) contain only A and T?
 - (d) do not contain C?
3. How many 4-element DNA sequences
 - (a) do not contain the base T?
 - (b) contain the sequence ACG?
 - (c) contain all four bases A, T, C, and G?
 - (d) contain exactly three of the four bases, A, T, C, and G?
4. How many bit strings of length 10 contain either five consecutive 0's or five consecutive 1's?
5. In how many ways can a set of two positive integers less than 100 be chosen?
6. A coin is flipped eight times where each flip comes up either heads or tails. How many possible outcomes
 - (a) are there in total?

- (b) contain exactly three heads?
 - (c) contain at least three heads?
 - (d) contain the same number of heads and tails?
7. How many strings of six lowercase letters from the alphabet $a - z$ contain
- (a) the letter a ?
 - (b) the letters a and b ?
 - (c) the letters a and b in consecutive positions with a preceeding b , with all the letters distinct?
 - (d) the letters a and b , with a somewhere to the left of b in the string, with all the letters distinct?
8. How many ways are there to select 12 countries in the United Nations to serve on a council if 3 are selected from a block of 45, 4 are selected from a block of 57, and the others are selected from the remaining 69 countries?
9. What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$?
10. Show that if n is a positive integer, then
- $$\binom{2n}{2} = 2\binom{n}{2} + n^2$$
- (a) using a combinatorial argument.
 - (b) by algebraic manipulation.