

Friday, Apr. 6

1. Use induction to prove that $1/(2n) \leq [1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)]/(2 \cdot 4 \cdot \dots \cdot 2n)$ whenever n is a positive integer.
2. How many different three-letter initials with none of the letters repeated can people have? (Note initials are only capital letters.)
3. How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?
4. Every student in a discrete math class is either a CS major or a Math major or is a double major in these two subjects. How many students are in the class if there are 38 CS majors (including joint majors), 23 Math majors (including joint majors), and 7 joint majors?
5. In how many different orders can five runners finish a race if no ties are allowed?

6. How many permutations of the letters ABCDEFGH contain
- (a) the string ED?
 - (b) the string CDE?
 - (c) the strings BA and FGH?
7. How many ways are there for a horse race with four horses to finish if ties are possible?
[Note: any number of the horses may tie]
8. How many different ways are there to choose a dozen donuts from the 21 varieties at a donut shop?
9. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants.
- (a) How many ways are there to choose a dozen croissants?
 - (b) How many ways are there to choose three dozen croissants?
 - (c) How many ways are there to choose two dozen croissants with at least two of each kind?
 - (d) How many ways are there to choose two dozen croissants with no more than two broccoli croissants?

10. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$, where x_i is an integer such that $x_i > 1$ for all i ?
11. How many strings of 20 decimal digits are there that contain two 0's, four 1's, three 2's, one 3, two 4's, three 5's, two 7's and three 9's?
12. How many ways are there to distribute 12 indistinguishable balls into six distinguishable bins?
13. How many ways are there to distribute 15 distinguishable objects into five distinguishable boxes so that the boxes have one, two, three, four and five objects in them, respectively? (You are free to pick which boxes get which numbers of balls).
14. How many different strings can be made from the letters in MISSISSIPPI, using all the letters?
15. How many different strings can be made from the letters in AARDVARK using all the letters, if all three A's must be consecutive?
16. How many different bit strings can be formed using six 1's and eight 0's?
17. From a deck of 52 cards, what is the probability that a five-card hand has five different kinds of cards (where "kinds of cards" refers to the 13 different kinds of cards in a deck of cards)?

18. A royal flush is the 10, jack, queen, king and ace all of the same suit. What is the probability that a five-card hand contains a royal flush?
19. A lottery is defined as follows. A player selects 7 numbers out of the numbers 1 through 80. Then the lottery computer randomly selects 11 numbers from the numbers 1 through 80. What is the probability that the 7 numbers the player selected are all from the 11 numbers the computer selected?
20. Suppose 50 people enter a contest.
- (a) What is the probability that Jeff, Mary, Xiaowei and Abrita win first, second, third and fourth, in that order, if no one can win more than one prize?
 - (b) What is the probability that Jeff, Mary, Xiaowei and Abrita win first, second, third and fourth, in that order, if winning more than one prize is allowed (names go back in after drawn)?
21. Which is more likely: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled?