Due Date: Thursday, Feb 2, 2012

1 Growth of functions and asymptotic behaviour (20 points)

In each of the following cases, rank the functions by order of their growth.

- $2^{\log n}, (\log n)^{\log n}, e^n, 4^{\log n}, n!, \sqrt{\log n}$
- $(\frac{3}{2})^n, n^3, (\log n)^2, \log(n!), 2^{2^n}, n^{\frac{1}{\log n}}$

2 More on asymptotic behaviour (20 points)

Show that, if c is a positive real number then $g(n) = 1 + c + c^2 + c^3 + c^4 + ...c^n$ is :

- $\Theta(1)$ if $c < 1$
- $\Theta(n)$ if $c = 1$
- $\Theta(c^n)$ if $c > 1$

3 Solving reccurences (20 points)

Solve the following recurrences by expanding the terms and give a Θ bound for each of them.

- $T(n) = 5T(n/4) + n$
- $T(n) = 9T(n/3) + n^2$
- $T(n) = T(\sqrt{n}) + 1$
- $T(n) = T(n-1) + n^c$

4 Comparing diffrent algorithms (20 points)

Suppose you are choosing between the following three algorithms:

- Algorithm A solves problems by dividing them into five subproblems of half size each and recursively solving each of the subproblem, and then combining the solutions in linear time
- Algorithm B solves problems of size n by recursively solving the two subproblems of size $n-1$ and then combining the solutions in constant time.
- Algorithm C solves the problems oif size n by dividing them into nine subproblems of size $n/3$ and recursively solving each subproblem and then combining the solutions into $O(n^2)$ time.

What are the running times of each of these algorithms and which would you choose?

5 Inversions (20 points)

For a sequence of natural numbers $S = \langle a_1, \ldots, a_n \rangle$, an *inversion* is a pair of elements (a_i, a_j) such that $i < j$ but $a_i > a_j$. For example, the inversions in sequence $\langle 1, 4, 2, 3 \rangle$ are $(4, 2)$ and $(4, 3)$; 4 appears before both 2 and 3. Given a sequence S of n natural numbers, describe an $O(n \log n)$-time algorithm to count the number of inversions in S. Prove its correctness and running time.
(**Hint:** *Modify a sorting algorithm.*)